EE431 Test II: Student Objectives Review Sheet

I. **DTFT**
- Given either x[n] or $X(e^{j\omega})$, find the other
- Explain what a time shift in x[n] does to $X(e^{j\omega})$
- Explain what addition, multiplication, and convolution of two time-domain signals does in the frequency-domain
- Explain what addition, multiplication, and convolution of two frequency-domain signals does in the time-domain
- Compute the energy density of a signal given the DTFT

II. **DFT**
- Describe how X[k] and $X(e^{j\omega})$ are related
- Given either X[k] or x[n], find the other
- Describe the symmetries of X[k]
- Compute the energy density of a signal given the DFT

III. **Z Tranforms**
- Given either X(z) or x[n], find the other (methods: long division, PDF, Matlab filter or Matlab residuez).
- If going from x[n] to X(z), find the ROC

IV. **Z Tranform & Block Diagrams**
- Given a block diagram, find the transfer function H(z)
- Tweak: find the system's h[n], or the response to a particular given x[n]
  find the system's difference equation

You may bring
- Calculator
- 1 3x5 notecard, both sides, your own work

I will provide
- The accompanying transform tables sheet

Common DTFT Pairs

x[n]	$X(e^{j\omega})$
$\delta[n]$	1
1	$2\pi\sum_{k=-\infty}^{\infty}\delta(\omega+2\pi k)$
$u[n]$	$\frac{1}{1-e^{-j\omega}}+\pi\sum_{k=-\infty}^{\infty}\delta(\omega+2\pi k)$
e^{j3n}	$2\pi\sum_{k=-\infty}^{\infty}\delta(\omega-3+2\pi k)$
$a^n u[n]$	$\frac{1}{1-a\,e^{-j\omega}}$

DTFT Properties

Property name	x[n], h[n]	$X(e^{j\omega})$, $H(e^{j\omega})$
Linearity	a x[n] + b h[n]	$a\,X(e^{j\omega}) + b\,H(e^{j\omega})$
Time-shifting	x[n-3]	$e^{-j\omega 3}\,X(e^{j\omega})$
Frequency-shift	e^{j3n} x[n]	$X(e^{j(\omega-3)})$
$\frac{d}{d\omega}(\ )$	-j n x[n]	$\frac{d}{d\omega}X(e^{j\omega})$
convolution	x[n] * h[n]	$X(e^{j\omega})\,H(e^{j\omega})$
multiplication	x[n] h[n]	$\frac{1}{2\pi}\int_{-\pi}^{\pi}X(e^{j\theta})\,H(e^{j(\omega-\theta)})d\theta$

DTFT Symmetries for real x[n]

$x[n] = x_{even}[n] + x_{odd}[n]$ $x_{even}[n]$ $x_{odd}[n]$	$X(e^{j\omega}) = X_{re}(e^{j\omega}) + j\,X_{im}(e^{j\omega})$ $X_{re}(e^{j\omega})$ $j\,X_{im}(e^{j\omega})$
	$\lvert X(e^{j\omega})\rvert$ and $X_{re}(e^{j\omega})$ are even from $-\pi$ to π $\angle X(e^{j\omega})$ and $X_{im}(e^{j\omega})$ are odd from $-\pi$ to π

DFT Properties

Property name	x[n], h[n]	X[k], H[k]
Linearity	a x[n] + b h[n]	a X[k] + b Y[k]
Time-shifting	$x[<n-3>_N]$	$e^{\frac{-j2\pi 3k}{N}}X[k]$
Frequency-shift	$e^{\frac{j2\pi 3n}{N}}x[n]$	$X[<k-3>_N]$
convolution	x[n] * h[n] $\sum_{m=0}^{N-1}x[m]h[\langle n-m\rangle_N]$	X[k]H[k]
multiplication	x[n] h[n]	$\frac{1}{N}\sum_{m=0}^{N-1}G[m]H[\langle k-m\rangle_N]$

DFT Symmetries for real x[n]

$x[n] = x_{even}[n] + x_{odd}[n]$ $x_{pcs}[n]$ $x_{pca}[n]$	$X[k] = X_{pcs}[k] + j\,X_{pca}[k]$ $X_{re}[k]$ $j\,X_{im}[k]$
	$\lvert X[k]\rvert$ and $X_{re}[k]$ pcs from 0 to N-1 $\angle X[k]$ and $X_{im}[k]$ pca from 0 to N-1

Common Z Transform Pairs

x[n]	X(z)	ROC
$\delta[n]$	1	All z
$u[n]$	$\frac{1}{1-z^{-1}}$	$\|z\| > 1$
$a^n u[n]$	$\frac{1}{1-a\,z^{-1}}$	$\|z\| > \|a\|$
$-(a^n) u[-n-1]$	$\frac{1}{1-a\,z^{-1}}$	$\|z\| < \|a\|$
$(n+1)\,a^n u[n]$	$\frac{1}{(1-a\,z^{-1})^2}$	$\|z\| < \|a\|$
$a^n \cos(\omega_o n)\, u[n]$	$\frac{1-(a\cos\omega_o)z^{-1}}{1-(2a\cos\omega_o)z^{-1}+a^2z^{-2}}$	$\|z\| > a$
$a^n \sin(\omega_o n)\, u[n]$	$\frac{(a\sin\omega_o)z^{-1}}{1-(2a\cos\omega_o)z^{-1}+a^2z^{-2}}$	$\|z\| > a$

Z Transform Properties

Property name	x[n], h[n]	X(z), H(z)	ROC: R_x, R_h
Conjugation	$x^*[n]$	$X^*(z^*)$	R_x
Time-reversal	$x[-n]$	$X(z^{-1})$	$1/R_x$
Linearity	$a\,x[n] + b\,h[n]$	$a\,X(z) + b\,H(z)$	Intersection of R_x, R_h
Time-shifting	$x[n-3]$	$z^{-3}\,X(z)$	R_x except possibly z=0 or ∞
Mult. by exponential	$a^n x[n]$	$X(z/a)$	$\|a\|\,R_x$
Differentiation of X(z)	$n\,x[n]$	$-z\frac{dX(z)}{dz}$	R_x except possibly z=0 or ∞
Convolution	$x[n] * h[n]$	$X(z)\,H(z)$	Intersection of R_x, R_h
Multiplication	$x[n]\,h[n]$	$\frac{1}{2\pi j}\oint_C X(v)H(\frac{z}{v})v^{-1}dv$	Union of R_x, R_h