Notation of Z transforms

In general, Z transforms are rational polynomials, and there are 4 ways to write them.

	Polynomial in z^{-1}	Polynomial in z
Unfactored	for example $$\frac{⑥}{①+3z^{-1}+2z^{-2}}$$ in general $$\frac{\textcircled{p_0}+p_1z^{-1}+\cdots+p_Mz^{-M}}{\textcircled{d_0}+d_1z^{-1}+\cdots+d_Nz^{-N}}$$ The correct form requires a circled coefficient not be multiplied by a power of z. Left-most denominator coefficient is 1.	for example e.g. $$z^2\frac{⑥}{z^2+3z+②}$$ in general $$z^{(N-M)}\frac{p_0z^M+p_1z^{M-1}+\cdots+\textcircled{p_M}}{d_0z^N+d_1z^{N-1}+\cdots+\textcircled{d_N}}$$ The correct form requires a circled coefficient not be multiplied by a power of z. Left-most denominator power of z coeff = 1.
Factored	for example $$6\frac{1}{(①+2z^{-1})(①+z^{-1})}$$ in general $$\frac{p_0}{d_0}\cdot\frac{\prod_{k=1}^{M}(①-\xi_kz^{-1})}{\prod_{k=1}^{N}(①-\lambda_kz^{-1})}$$ The correct form requires a circled numbers to be exactly 1.	for example $$6z^2\frac{1}{(\textcircled{z}+2)(\textcircled{z}+1)}$$ in general $$\frac{p_0}{d_0}z^{N-M}\cdot\frac{\prod_{k=1}^{M}(\textcircled{z}-\xi_k)}{\prod_{k=1}^{N}(\textcircled{z}-\lambda_k)}$$ The correct form requires a circled variable to be exactly z with coefficient of 1.

Notes:

- All the following use the same example, written in different ways.
- The general examples all have a numerator order of M and a denominator order of N (if they represent systems we could say they represent IIR systems of order N, or for N=0, FIR systems of order M)
- For the factored cases
  - ξ are the zeros of X(z) (roots when setting the numerator = 0)
  - λ are the poles of X(z) (roots when setting the denominator = 0)
  - additional (N-M) zeros at z=0 if N>M
  - additional (M-N) poles at z=0 if M>N