1. Find the DTFT of x[n]=

a. $6\,\delta[n] - 2\,\delta[n+1]$

by tables is

$$\boxed{6-2e^{j\omega}}$$

b. $2^{-|n|}$

$2^{-|n|}$ by def. of DTFT is

$$\sum_{n=-\infty}^{\infty} 2^{-|n|}e^{-j\omega n} = \sum_{n=-\infty}^{0} 2^{n}e^{-j\omega n} + \sum_{n=0}^{\infty} 2^{-n}e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{0} (2e^{-j\omega})^{n} + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}e^{-j\omega}\right)^{n}$$

$$= \sum_{n=0}^{\infty} (2e^{-j\omega})^{-n} + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}e^{-j\omega}\right)^{n} = \sum_{n=0}^{\infty} \left(\tfrac{1}{2}e^{j\omega}\right)^{n} + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}e^{-j\omega}\right)^{n}$$

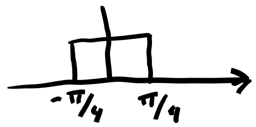
recall if $y = \sum_{n=0}^{\infty} a^{n} = a^{0} + a^{1} + a^{2} + \ldots$

$$\begin{array}{rl} ay & = a^{1} + a^{2} + \ldots \\ \hline y(1-a) & = a^{0} = 1 \end{array} \Rightarrow y = \frac{1}{1-a}$$

$$= \boxed{\frac{1}{1-\frac{1}{2}e^{-j\omega}} + \frac{1}{1-\frac{1}{2}e^{j\omega}}}$$

2. Find the IDTFT of

a. $X(e^{j\omega}) = \begin{cases} 1, & -\frac{\pi}{4} \le \omega \le \frac{\pi}{4} \\ 0, & \text{elswhere over } -\pi < \omega < \pi \end{cases}$



by defn $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}\,d\omega$

$$= \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{j\omega n}\,d\omega = \frac{1}{2\pi}\,\frac{1}{jn}\,e^{j\omega n}\Big]_{\omega=-\pi/4}^{\pi/4}$$

$$= \frac{1}{j2\pi n}\left[e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n}\right] \text{ but } \sin\theta = \frac{1}{j2}\left[e^{j\theta} - e^{-j\theta}\right]$$

$$= \boxed{\frac{1}{\pi n}\sin\left(\frac{\pi}{4}n\right)}$$

b. $X(e^{j\omega}) = 6\cos(2\omega)$

$6\cos(2\omega) = 3[e^{j2\omega} + e^{-j2\omega}]$ from Euler's $\cos\theta = \frac{1}{2}[e^{j\theta} + e^{-j\theta}]$

$\boxed{x[n] = 3\delta[n-2] + 3\delta[n+2]}$ from tables

3. For some $x[n]$, $X[k] = [\underset{\uparrow}{6}\ 2\ 5\ 5\ 2]$

a. How long is x[n]?

Same as $X[k]$, $\boxed{5}$ (ie $N=5$)

b. What is $\sum x[n]$?

$X[k=0] = \sum x[n] = \boxed{6}$

c. What sample in the DFT corresponds to frequency $\omega = 0.4\pi$?

$\frac{N}{2\pi} = \frac{k}{\omega}$ so $\frac{5}{2\pi} = \frac{k}{\frac{2}{5}\pi} \Rightarrow 2\pi k = 2\pi \Rightarrow \boxed{k=1}$

d. If the sampling frequency from which x[n] is taken is 60Hz, which sample of the DFT corresponds to 24Hz?

$\frac{N}{fs} = \frac{k}{f}$ so $\frac{5}{60} = \frac{k}{24}$, $\boxed{k=2}$

e. What kind of symmetry does x[n] (not X[k]) have?

Since $X[k]$ is real, $x[n]$ must be $\boxed{\text{periodic symmetric}}$

4. Find the z transform of $x[n] = (-0.3)^n \mathrm{u}[n-2]$

Tables

$(-0.3)^n u[n-2]$

$= (-0.3)^{n-2}(0.3)^2 u[n-2]$

$= 0.09(-0.3)^{n-2} u[n-2]$

$\Leftrightarrow \boxed{\frac{0.09}{1+0.3z^{-1}} z^{-2}}$

Defn

$\sum_{n=-\infty}^{\infty} x[n] z^{-n}$

$= \sum_{n=-\infty}^{\infty} (-0.3)^n u[n-2] z^{-n}$

$= \sum_{n=2}^{\infty} (-0.3z^{-1})^n$

$= \frac{(-0.3z^{-1})^2}{1+0.3z^{-1}}$

$= \boxed{\frac{0.09z^{-2}}{1+0.3z^{-1}}}$

aside:

$Y = \sum_{n=2}^{\infty} a^n$

$Y = a^2 + a^3 + a^4 + \cdots$

$-[aY = a^3 + a^4 + a^5 + \cdots]$

$Y - aY = a^2$

$Y(1-a) = a^2$

$Y = \frac{a^2}{1-a}$

5. Find the inverse z transform of $\frac{z+z^{-1}}{1+0.8z^{-1}-0.2z^{-2}}$

1) standard form $z\left[\frac{1+z^{-2}}{1+0.8z^{-1}-0.2z^{-2}}\right]$

2) proper fraction (do poly division and <u>reverse</u> the coefficients)

$$-0.2z^{-2}+0.8z^{-1}+1 \overline{\big)\, z^{-2}+1}$$

quotient: -5

$$\frac{z^{-2}-4z^{-1}-5}{4z^{-1}+6}$$

$$\Rightarrow -5+\frac{4z^{-1}+6}{1+0.8z^{-1}-0.2z^{-2}}$$

3) Fraction and PFD

$$z\left[-5+\frac{4z^{-1}+6}{(1+z^{-1})(1-0.2z^{-1})}\right]=z\left[-5+\frac{\frac{4(-1)+6}{1+0.2}=\frac{5}{3}}{1+z^{-1}}+\frac{\frac{20+6}{6}=\frac{13}{3}}{1-0.2z^{-1}}\right]$$

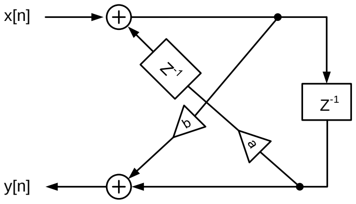
4) Inverse z-transform by tables

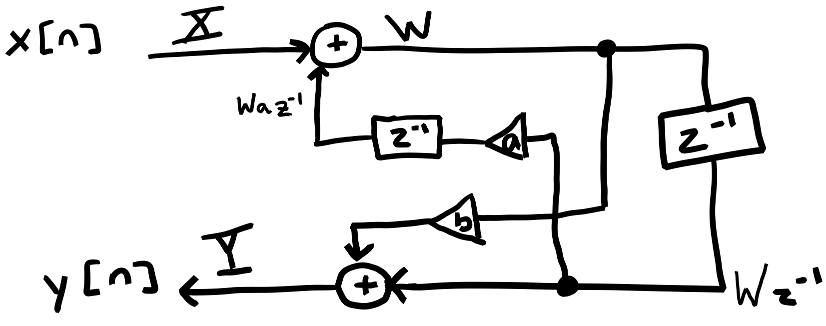
$$\left[-5\delta[n]+\frac{5}{3}(-1)^n u[n]+\frac{13}{3}\left(\frac{1}{5}\right)^n u[n]\right]\Bigg|_{n\rightarrow n+1}$$

$$=\boxed{-5\delta[n+1]+\left[\frac{5}{3}(-1)^{n+1}+\frac{13}{3}\left(\frac{1}{5}\right)^{n+1}\right]u[n+1]}$$

trouble w/ factoring $0.2z^{-2}+0.8z^{-1}+1$ by inspection? Do this:
mult by z^2 ⇒ $0.2+0.8z+z^2=0$.
quad formula: $a=1, b=0.8, c=0.2$ ⇒ $\frac{-0.8\pm\sqrt{0.64+0.8}}{2}=-0.4\pm\frac{1}{2}\sqrt{1.44}=-0.4\pm\frac{1}{2}\cdot1.2=-0.4\pm0.6=0.2,-1$
so $(1-0.2z^{-1})(1+z^{-1})$

6. Find H(z) of this system





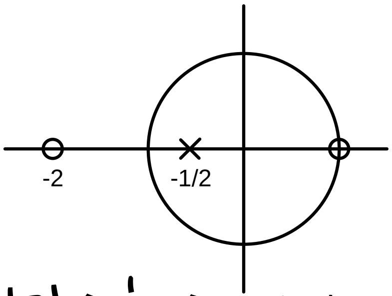
At top ⊕: $W=X+Waz^{-2} \Rightarrow W(1-az^{-2})=X \Rightarrow W=\frac{X}{1-az^{-2}}$

At bot ⊕: $Y=z^{-1}W+bW \Rightarrow W(b+z^{-1})=Y \Rightarrow W=\frac{Y}{b+z^{-1}}$

Set equal

$$\frac{X}{1-az^{-2}}=\frac{Y}{b+z^{-1}} \Rightarrow H(z)=\frac{Y}{X}=\boxed{\frac{b+z^{-1}}{1-az^{-2}}}$$

7. Given



a. Is the filter BIBO stable?

Causal => ROC is $|z| > \frac{1}{2}$ => includes unit circle =>
$\boxed{\text{BIBO Stable}}$

b. FIR or IIR?

poles at places other than origin => $\boxed{\text{IIR}}$

c. Linear phase?

IIR so $\boxed{\text{Cannot be lin phase}}$

d. LP, HP, BP, BS, or other?

Pole close to $\omega = \pi$ = high frequency boost
Zero close to $\omega = 0$ = low frequency cut
} $\boxed{\text{high pass filter}}$

e. What is the response to x[n] = 5?

=> response to $\omega = 0$ signal (unchanging)
=> $\boxed{0}$ since a zero at $\omega = 0$

8. Using Matlab's filterDesigner, find h[n] of a linear phase filter used in a system with a 1kHz sampling frequency that passes frequencies ≤ 100Hz with <u>about</u> a ±10% variance from unity gain, and attenuates frequencies ≥ 250Hz by <u>about</u> 90%.

filter Designer

Loupass	Hz	
FIR equiripple	$F_s = 1000$	Linear
Min order	$F_p = 100$	Dpass = 0.1
	$F_s = 250$	Dstep = 0.1

Menu Analysis
=> show
Coefficients

$$h[n] = 0.09549\ \delta[n] + 0.30917\ \delta[n-1] + 0.3159\ \delta[n-2] + 0.30917\ \delta[n-3] + 0.09549\ \delta[n-4]$$