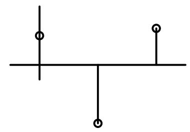
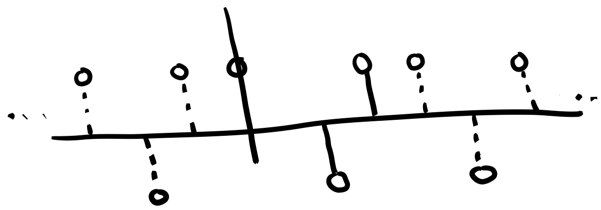
1. What **symmetry**, if any, does  (zero elsewhere) have?

Not conj symmetric or antisym since it has non zero for $n > 0$ but not $n < 0$.
See if periodic symm by forming periodic extension



$\boxed{\text{neither sym nor anti-sym nor periodic sym nor antisym}}$

Side note: This is a periodic symmetric waveform time-advanced by 1/2 sample, so all samples of its DFT will have a linearly-increasing phase of

$$e^{-j2\pi(-1/2)k/N} = e^{j\pi k/3} = (60^\circ)k$$

2. Find the periodic, conj-sym part of x[n] = [6, 1+j, 6+j]
$\uparrow$

$x_{pcs}[n] = \frac{1}{2}\{x[n] + x^*[\langle -n \rangle_N]\}$

to find $x^*[\langle -n \rangle_N]$ form periodic extension of $x[n] \Rightarrow [\ldots 6, 1+j, 6+j, \underset{\uparrow}{6}, 1+j, 6+j, 6 \ldots]$

reverse to find $x[\langle -n \rangle_N] \Rightarrow [\ldots 6, 6+j, 1+j, \underset{\uparrow}{6}, 6+j, 1+j, 6 \ldots]$

take conjugate and limit to $0 \le n < N \Longrightarrow [\underset{\uparrow}{6}, 6-j, 1-j]$

$x_{pcs}[n] = \frac{1}{2}\{[6, 1+j, 6+j] + [6, 6-j, 1-j]\}$

$= \frac{1}{2}\{[12, 7, 7]\}$

$= \boxed{[6 \quad 3.5 \quad 3.5]}$

3. What is the digital frequency $0 \le \omega \le \pi$ of a 100Hz cos wave sampled at fs = 75Hz?

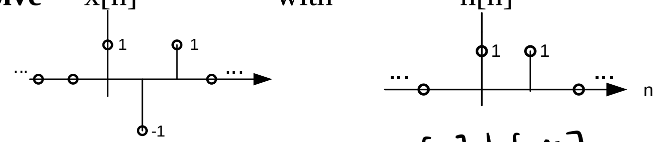
$x(t) = \cos(2\pi 100 t)$, $f_s = 75 \Rightarrow T_s = 1/75$

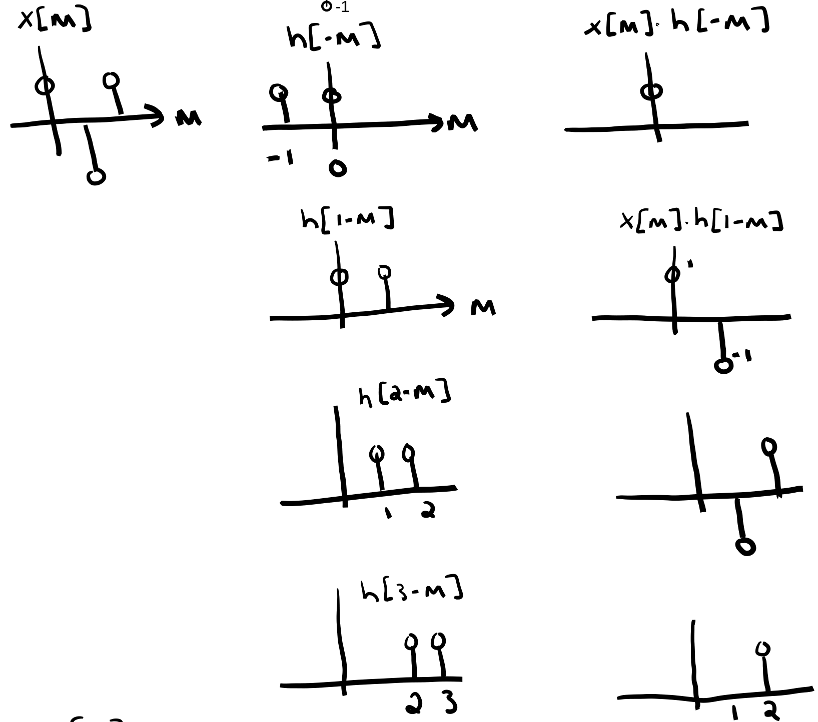
$x[n] = x(nT_s) = \cos(2\pi 100n/75) = \cos(2\pi \, 4/3 \, n)$

$= \cos(8\pi/3 \, n) = \cos(8\pi/3 \, n - 2\pi n) = \cos(2/3 \, \pi n)$

$\boxed{\omega = 2/3 \pi}$

4. Graphically **convolve** x[n] with h[n]



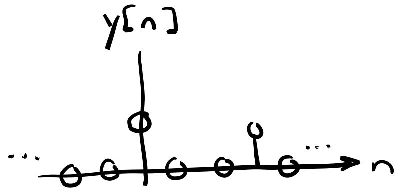


$y[0] = 1$

$y[1] = 1 - 1 = 0$

$y[2] = -1 + 1 = 0$

$y[3] = 1$



5. **Given** the DE $y[n] + \frac{1}{4}y[n-1] = x[n] + \frac{1}{2}x[n-1]$ and $x[n] = 2\left(\frac{1}{2}\right)^n u[n]$

Find y[n]

input: $X(z) = \dfrac{2}{1 - \frac{1}{2}z^{-1}}$

system: $Y(z) + \frac{1}{4}z^{-1}Y(z) = X(z) + \frac{1}{2}z^{-1}X(z)$

$Y(z)\left[1 + \frac{1}{4}z^{-1}\right] = X(z)\left[1 + \frac{1}{2}z^{-1}\right]$

$H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{1 + \frac{1}{2}z^{-1}}{1 + \frac{1}{4}z^{-1}}$

$Y(z) = X(z)H(z)$

$= \dfrac{2\left(1 + \frac{1}{2}z^{-1}\right)}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{4}z^{-1}\right)}$

$= \dfrac{A}{1 - \frac{1}{2}z^{-1}} + \dfrac{B}{1 + \frac{1}{4}z^{-1}}$

$A = \dfrac{2\left(1 + \frac{1}{2}\cdot 2\right)}{1 + \frac{1}{4}\cdot 2} = \dfrac{4}{3/2}\cdot\dfrac{2}{2} = \dfrac{8}{3}$

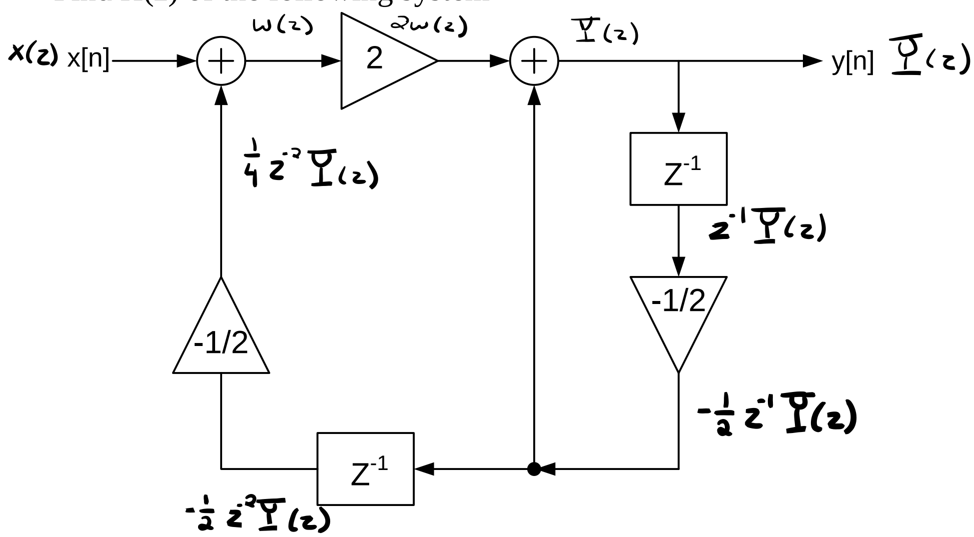
$B = \dfrac{2\left(1 + \frac{1}{2}(-4)\right)}{1 - \frac{1}{2}(-4)} = \dfrac{2(-1)}{1+2} = -\dfrac{2}{3}$

$Y(z) = \dfrac{8/3}{1 - \frac{1}{2}z^{-1}} - \dfrac{2/3}{1 + \frac{1}{4}z^{-1}}$

important! o/w grows for $n < 0$

$$\boxed{y[n] = \left[\frac{8}{3}\left(\frac{1}{2}\right)^n - \frac{2}{3}\left(-\frac{1}{4}\right)^n\right]u[n]}$$

6. Find H(z) of the following system



First create new variables after each summer, if neccessary
Then write eqns at summers + simplify to remove added variables

$W(z) = X(z) + \frac{1}{4}z^{-2}Y(z)$

$Y(z) = 2W(z) - \frac{1}{2}z^{-1}Y(z) \Rightarrow Y(z)\left[\frac{1+\frac{1}{2}z^{-1}}{2}\right] = W(z)$

$Y(z)\left[\frac{1}{2} + \frac{1}{4}z^{-1}\right] = X(z) + \frac{1}{4}z^{-2}Y(z)$

$Y(z)\left[\frac{1}{2} + \frac{1}{4}z^{-1} - \frac{1}{4}z^{-2}\right] = X(z)$

$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{\frac{1}{2}+\frac{1}{4}z^{-1}-\frac{1}{4}z^{-2}}$ in standard form $= \boxed{\frac{2}{1+\frac{1}{2}z^{-1}-\frac{1}{2}z^{-2}}}$

7. **Find** the impulse response of the above system.

$$H(z) = \frac{2}{(1+z^{-1})(1-\frac{1}{2}z^{-1})} = \frac{\frac{2}{1+\frac{1}{2}} = 4/3}{1+z^{-1}} + \frac{\frac{2}{1+2} = 2/3}{1-\frac{1}{2}z^{-1}}$$

$$\boxed{h[n] = \left[\tfrac{4}{3}(-1)^n + \tfrac{2}{3}\left(\tfrac{1}{2}\right)^n\right]u[n]}$$