Last Class

Geometric interpretation of $H(z)$

$$|H(e^{j\omega})| = \frac{\prod(\text{distances b/n } \omega \text{ and zeros})}{\prod(\text{distances b/n } \omega \text{ and poles})}$$

Obj • zero phase / linear phase
• 4 Types of FIR filters

Review FIR filter

$$H(z) = \sum_{k=0}^{M} a_k z^{-k} \quad \text{ex} \quad \boxed{1 + 2z^{-1} + z^{-2}} \quad M=2$$

$$= 1 + \frac{2}{z} + \frac{1}{z^2}$$

zeros

$$1 + 2z^{-1} + z^{-2} = 0$$

$$z^2 + 2z + 1 = 0$$

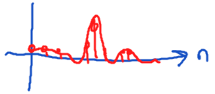
} factor or quadratic eqn to find zeros

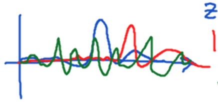
poles of FIR filter

will be at origin

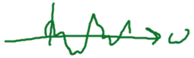
Zero Phase Filter

Every system $H(e^{j\omega})$ has a phase function $\theta(\omega) = \angle H(e^{j\omega})$





zero phase filter
linear phase filter $\theta(\omega) = k\omega$ (delay)
non linear phase filter $\theta(\omega)$



Zero phase filters

$\theta(\omega) = 0 \Rightarrow H(e^{j\omega})$ purely real
$\Rightarrow h[n]$ even

Good ① Does not change shape of a signal
② Does not delay signal

$$\tau_g = -\frac{d\theta(\omega)}{d\omega} \quad \text{so if } \theta(\omega) = 0, \; -\frac{d\theta}{d\omega} = 0 = \tau_g$$

Bad ① Non-causal $\Rightarrow h[n]$ even

Linear Phase Filter ⇔ FIR

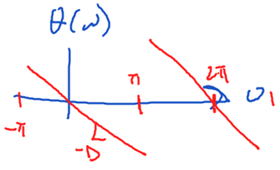
Good ① Does not change shape signal, like phase
② Is causal

To make Delay a zero phase filter until causal

$$x_{\text{Lin phase}}[n] = x_{\text{zero phase}}[n - D]$$

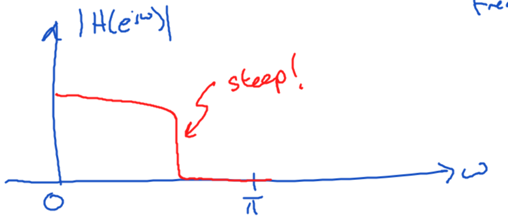
$$X_{\text{lin phase}}(e^{j\omega}) = X_{ZP}(e^{j\omega}) \cdot e^{-j\omega D}$$

$$\theta_{\text{lin phase}}(e^{j\omega}) = \cancel{\theta_{ZP}(\omega)}^{\,0} + -\underline{\omega D}$$



Non linear Phase Filter

Good IIR $\Leftrightarrow$ non linear phase $\Rightarrow$ powerful in freq domain



Bad "Messes up" signal in time domain

New Notation

IIR $\quad H(z) = \dfrac{\sum_{k=0}^{M} a_k z^{-k}}{\sum_{k=0}^{N} b_k z^{-k}}$ order $\underline{\underline{N}}$ ex $\dfrac{2+z^{-1}}{1+3z^{-1}-2z^{-2}}$

FIR $\quad H(z) = \sum_{k=0}^{N} c_k z^{-k}$ order $\underline{\underline{N}}$ ex $1+\frac{1}{2}z^{-1}+z^{-2}$

FIR since length $h[n]=3$
linear phase since $h[n]$ is shifted even
$[1 \quad \frac{1}{2} \quad 1]$
lin. phase
zero phase

Ways To classify Systems

Mag / Gain Response $\quad G(\omega) = |H(e^{j\omega})| = \begin{cases} LP \\ HP \\ \dots \end{cases}$

Phase Response $\quad \theta(\omega) = \angle H(e^{j\omega}) = \begin{cases} 0 \text{ phase} \\ \text{lin phase} \\ \text{nonl. phase} \end{cases}$

impulse response $\quad h[n]$ $\begin{cases} \text{FIR} = \text{linear phase} \begin{cases} \text{good time domain} \\ \text{bad freq domain} \end{cases} \\ \text{IIR} = \text{non linear phase} \begin{cases} \text{good freq domain} \\ \text{bad time domain} \end{cases} \end{cases}$