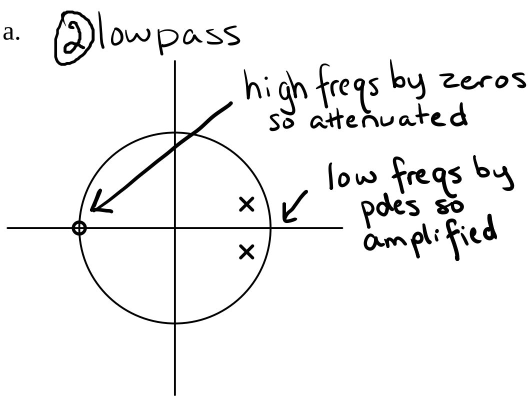
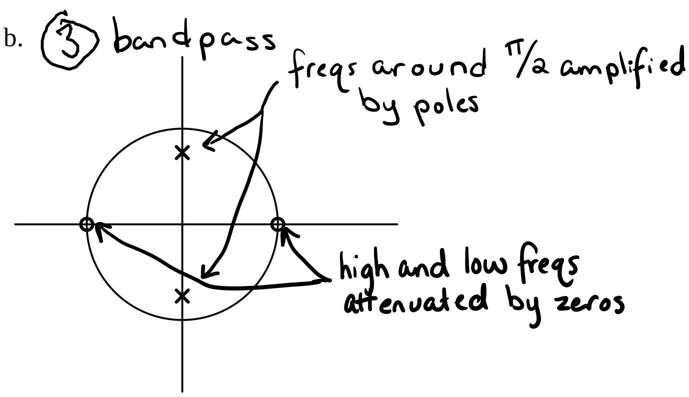
1. For each of the following causal filters, choose one of the following 4 descriptions:
   1. Unstable
   2. Lowpass
   3. Bandpass
   4. Highpass

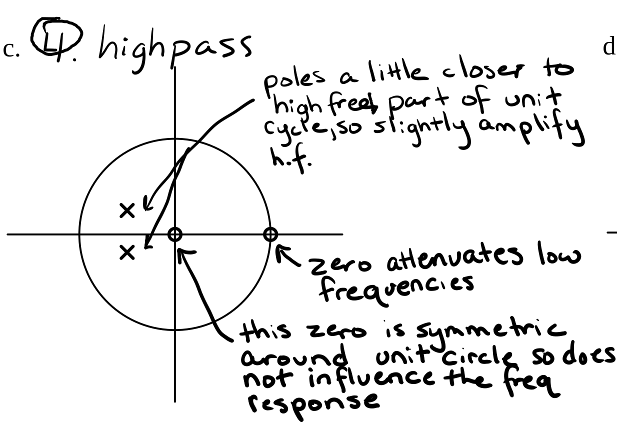
a.



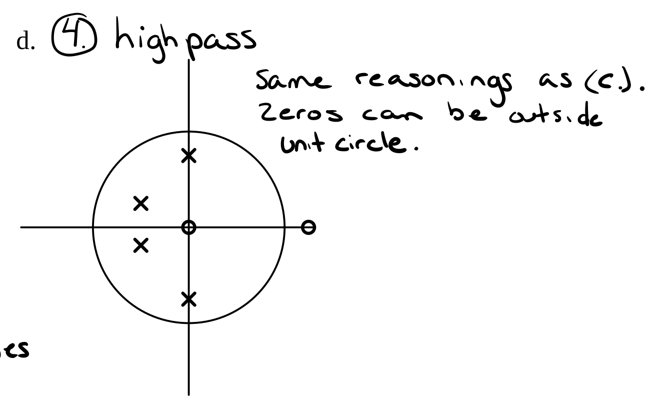
b.



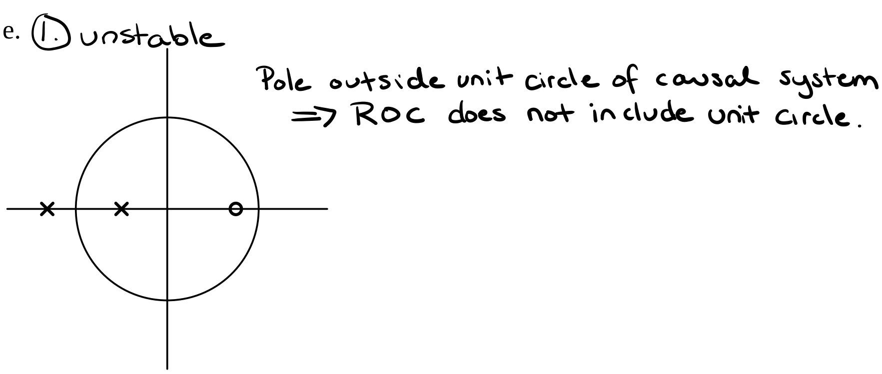
c.



d.



e.



2. A FIR LTI system is described by

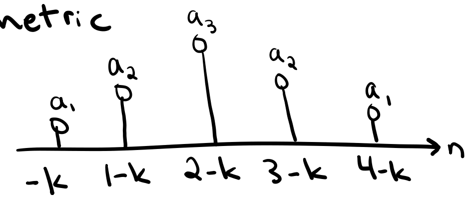
$$y[n] = a_1\, x[n+k] + a_2 x[n+k-1] + a_3 x[n+k-2] + a_2 x[n+k-3] + a_1 x[n+k-4].$$

a. Find $H(e^{j\omega})$

$$= a_1 e^{j\omega k} + a_2 e^{j\omega(k-1)} + a_3 e^{j\omega(k-2)} + a_2 e^{j\omega(k-3)} + a_1 e^{j\omega(k-4)}$$

b. For what values of k will $H(e^{j\omega})$ be purely real?

A DTFT is real if the time-domain signal is conjugate symmetric



To be conj symmetric, $2-k=0$ $\Rightarrow$ $\boxed{k=2}$

A math justification is:

$$H(e^{j\omega}) = (a_1 e^{j\omega 2} + a_1 e^{-j\omega}) + (a_2 e^{j\omega} + a_2 e^{-j\omega}) + a_3$$

$$= 2a_1 \cos(2\omega) + 2a_2 \cos(\omega) + a_3$$

3. Given causal $H(z) = \frac{6+2z^{-1}}{4+kz^{-2}}$

Find the range of the k for which the system is stable.

Stable if |all poles| < 1

poles are at $\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{\pm\sqrt{-4\cdot 4\cdot k}}{8} = \pm\frac{1}{8}\sqrt{-16k} = \pm\frac{j}{2}\sqrt{k}$

$$\left|\pm\frac{j}{2}\sqrt{k}\right| = \left|\frac{j}{2}\right|\left|\sqrt{k}\right| = \frac{1}{2}\sqrt{|k|} < 1$$

$$= \sqrt{|k|} < 2$$

$$= |k| < 4$$

$$= \boxed{-4 < k < 4}$$

Check Matlab:

```
k= linspace(-10,10,500)
pole= sqrt(-16*k)'/8;
plot(k,abs(pole))
```