$$5\left(\frac{2}{3}\right)^n u[n] \rightarrow \boxed{H(z) = \frac{1}{1 - \frac{1}{2}z^{-1}}} \rightarrow y[n]$$

Find $y[n]$ after a long time using the steady-state method.

① Make $x[n] = Ae^{j\omega_0 n}$

$5\left(\frac{2}{3}\right)^n = Ae^{j\omega_0 n}$

$\left(\frac{2}{3}\right)^n = e^{j\omega_0 n}$ $\quad A = 5$ by inspection

$\ln\left(\frac{2}{3}\right) = j\omega_0$

$\omega_0 = \frac{\ln(2/3)}{j}$

② $y_{ss}[n] = x[n]\,H(e^{j\omega_0})$

$= 5\left(\frac{2}{3}\right)^n \left[\frac{1}{1 - \frac{1}{2}(e^{j\omega_0})^{-1}}\right]$

$= 5\left(\frac{2}{3}\right)^n \left[\frac{1}{1 - \frac{1}{2}\left(e^{j\frac{\ln(2/3)}{j}}\right)^{-1}}\right]$

$= 5\left(\frac{2}{3}\right)^n \left[\frac{1}{1 - \frac{1}{2}\left(\frac{2}{3}\right)^{-1}}\right]$

$= 5\left(\frac{2}{3}\right)^n \left[\frac{1}{1 - \frac{1}{2}\left(\frac{3}{2}\right)}\right]$

$= 5\left(\frac{2}{3}\right)^n \left[\frac{1}{1 - \frac{3}{4}}\right]$

$= 5\left(\frac{2}{3}\right)^n \left[\frac{1}{1/4}\right]$

$= 5\left(\frac{2}{3}\right)^n 4$

$= \boxed{20\left(\frac{2}{3}\right)^n}$