Read : 4.1 - 4.2.2

HW : None ! ☺

Obj : Chap 4 Overview
- DT Systems in freq domain
- Transfer Functions : 3 perspectives
- MATLAB

Overview Chap 4

Last Test Block : Signals in the freq domain
This Test Block : Systems in the freq domain

ω	DTFT	want
k	DFT	can compute
z	Z transform	block diagrams

$z \leftrightarrow e^{j\omega}$

Perspective #1: equivalent to time-domain DE

$\Rightarrow$ Complete description of system

Ex $\quad y[n] - \frac{1}{2}y[n-1] = 6x[n] + x[n-1]$

$$Y(z) - \frac{1}{2}Y(z)z^{-1} = 6X(z) + X(z)z^{-1}$$

$$Y(z)\left[1 - \frac{1}{2}z^{-1}\right] = X(z)\left[6 + z^{-1}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = \boxed{\frac{6+z^{-1}}{1-\frac{1}{2}z^{-1}}} \qquad z \leftrightarrow e^{j\omega}$$

$$\boxed{H(e^{j\omega}) = \frac{6+e^{-j\omega}}{1-\frac{1}{2}e^{-j\omega}}}$$

So: • $\underbrace{H(z), H(e^{j\omega})}_{\text{freq domain}} \longleftrightarrow \underbrace{\text{Diff Eqn}}_{\text{time domain}}$ directly (no tables)

• Given $X(z)$, DE $\longrightarrow Y(z) = H(z) \cdot X(z)$

⋮

Perspective #2 Impulse Response

"hit it with a hammer"

$\delta[n] \longrightarrow \boxed{\text{system}} \longrightarrow h[n]$

since any input $x[n]$ can be made by adding shifted & scaled $\delta[n]$

any output $y[n]$ can be found by " " " $h[n]$

aka convolution

$$y[n] = \sum_{m=-\infty}^{\infty} h[m]\, x[n-m]$$

Amazing fact $h[n] \Longleftrightarrow H(z)$ from perspective #1

Perspective #3 frequency response

eigenfunctions $\longrightarrow$ system $\longrightarrow$ $k \cdot$ eigenfunctio

↑ complex const nt

Ex 1: $e^{j\omega_0 n} \longrightarrow$ System $\longrightarrow \underbrace{H(e^{j\omega_0})}_{\text{complex \#}} \cdot e^{j\omega_0 n}$

Ex 2: $\cos(\omega_0 n) \longrightarrow$ system $\longrightarrow \underbrace{|H(e^{j\omega_0})|}_{\text{amplitude}} \cdot \cos(\omega_0 n + \underbrace{\angle H(e^{j\omega_0})}_{\text{phase}})$

amplitude function of ω_0

phase function of ω_0

Euler's Identities

$$e^{\pm j\omega} = \cos(\omega) \pm j\sin(\omega)$$

$$\cos\omega = \frac{e^{j\omega} + e^{-j\omega}}{2}$$

$$\sin\omega = \frac{e^{j\omega} - e^{-j\omega}}{j2}$$

Plot gain amplitude, phase vs ω

$\Rightarrow$ Bode Plot!

$$\left.\begin{array}{l} \text{Gain Function : } G(\omega) = 20\log_{10}|H(e^{j\omega})| \text{ dB} \\ \text{Phase Function : } \theta(\omega) = \angle H(e^{j\omega}) \text{ rads} \end{array}\right\} H(e^{j\omega})$$

Ex Find $G(\omega)$ if $H(e^{j\omega}) = 2e^{j\omega} + 2e^{-j\omega}$

$$G(\omega) = 20\log_{10}\left(\tfrac{1}{2}(4e^{j\omega} + 4e^{-j\omega})\right)$$

$$\boxed{20\log_{10}(4\cos\omega)}$$

Find $h[n]$ if $H(e^{j\omega}) = 2e^{j\omega} + 2e^{-j\omega}$

$$H(z) = 2z + 2z^{-1}$$

$$\boxed{h[n] = 2\delta[n+1] + 2\delta[n-1]}$$

MATLAB

freq response freqz(num, den, w)

Ex plot $G(\omega)$ of 4 point, causal, moving average filter

$$y[n] = \frac{x[n] + x[n-1] + x[n-2] + x[n-3]}{4}$$

$$Y(z) = \frac{1}{4}X(z) + \frac{1}{4}X(z)\cdot z^{-1} + \frac{1}{4}X(z)\cdot z^{-2} + \cdots$$

$$Y(z) = X(z)\left[\frac{1}{4} + \frac{1}{4}z^{-1} + \frac{1}{4}z^{-2} + \frac{1}{4}z^{-3}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = \boxed{\frac{1}{4} + \frac{1}{4}z^{-1} + \frac{1}{4}z^{-2} + \frac{1}{4}z^{-3}}$$

```
>> w = linspace(0, pi, 1000);
>> H = freqz([1/4, 1/4, 1/4, 1/4], 1, w);
>> plot(w, 20*log10(abs(H)));
>> plot(w, angle(H))
```

unwrap