

### Common Z Transform Pairs

$x[n]$	$X(z)$	ROC
$\delta[n]$	1	All $z$
$u[n]$	$\frac{1}{1 - z^{-1}}$	$ z  > 1$
$a^n u[n]$	$\frac{1}{1 - a z^{-1}}$	$ z  >  a $
$(n+1) a^n u[n]$	$\frac{1}{(1 - a z^{-1})^2}$	$ z  <  a $
$a^n \cos(\omega_0 n) u[n]$	$\frac{1 - (a \cos \omega_0) z^{-1}}{1 - (2a \cos \omega_0) z^{-1} + a^2 z^{-2}}$	$ z  > a$
$a^n \sin(\omega_0 n) u[n]$	$\frac{(a \sin \omega_0) z^{-1}}{1 - (2a \cos \omega_0) z^{-1} + a^2 z^{-2}}$	$ z  > a$

Common example: any finite length  $x[n]$  can be broken into sums of scaled, time-shifted  $\delta[n-n_0]$ , so its Z transform will be sums of similarly scaled  $z^{-n_0}$  (see time-shift property below), with a ROC everywhere except possibly  $z=0$  or  $z=\infty$  (evaluate  $z$  at these points to see)

### Z Transform Properties

Property name	$x[n], h[n]$	$X(z), H(z)$	ROC: $R_x, R_h$
Linearity	$a x[n] + b h[n]$	$a X(z) + b H(z)$	Intersection of $R_x, R_h$
Time-shifting	$x[n-3]$	$z^{-3} X(z)$	$R_x$ except possibly $z=0$ or $\infty$ (evaluate $z$ at these points to see)
Differentiation of $X(z)$	$n x[n]$	$-z \frac{dX(z)}{dz}$	$R_x$ except possibly $z=0$ or $\infty$
Convolution	$x[n] * h[n]$	$X(z) H(z)$	Intersection of $R_x, R_h$