1. Consider the following finite-length sequences with N=8 defined for $0 \le n \le 7$:

- $x_1[n]$ = [1 1 1 0 0 0 1 1]
- $x_2[n]$ = [1 1 0 0 0 0 -1 -1]
- $x_3[n]$ = [0 1 1 0 0 0 -1 -1]
- $x_4[n]$ = [0 1 1 0 0 0 1 1]

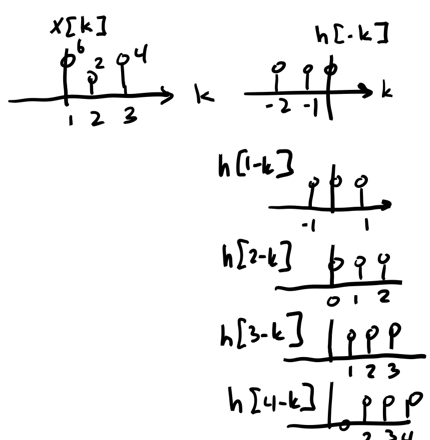
Without explicitly calculating DFTs:
a. Which have purely real DFTs (i.e. which are purely pcs?)
b. Which have purely imaginary DFTs (i.e. which are purely pca)?
c. Which have X[k=0] = 0? (Hint: last PS asked what is X[k=0])

a) $\boxed{x_1, x_4}$ are pcs → real DFT. Why? x_1: (1 1 1 0 0 0 1 1) 1 1 1 0 0 0 1 1 (1 1 ...) replicate etc. replicate

b) $\boxed{x_3}$ is pca → imag DFT. Why? x_3: (0 1 1 0 0 0 -1 -1) 0 1 1 0 0 0 -1 -1 (0 1 1 0 ...) etc.

c) $\boxed{x_2, x_3}$ have $X[k=0] = \sum_{n=0}^{N-1} x[n] = 0$

2. Graphically find this circular convolution: [6 2 4] ⊛$_3$ [1 1 1] if both sequences are finite length starting at n=0.



$y[0] = 6$

$y[1] = 6+2 = 8$

$y[2] = 6+2+4 = 12$

$y[3] = 2+4 = 6$

$y[4] = 4$

$x[n] * h[n] = [6\ 8\ 12\ 6\ 4]$

$x[n] \circledast_3 h[n] = [12\ 12\ 12]$

$\boxed{[12\ 12\ 12]}$

3. If $x[n] \circledast_5 h[n]$ = [8 2 9 10 -4] (and the answer starts at n=0 as usual), can you find $x[n] \circledast_4 h[n]$? If so, what is it?

No; you cannot tell. (To see why, let the linear convolution be [x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8], and compare the 5 and 4 length time-aliased sequences).

4. To find the **linear** convolution of x[n] = [1 2 3] with h[n] = [1 1 1 1 1] using DFTs, how many zeros must you end-pad x and h by? i.e. in Matlab, how many zeros would you have to end-pad x and h by before evaluating `>>ifft(fft(x) .* fft(h))`

Linear convolution has length $N_x+N_h-1 = 3+5-1 = 7$ so pad each input to length 7.

x = [1 2 3 $\underbrace{0\ 0\ 0\ 0}_{\text{4 zeros}}$], h=[1 1 1 1 1 $\underbrace{0\ 0}_{\text{2 zeros}}$]

5. If you are only interested in graphing the energy density of a signal (i.e. $|X[k]|^2$) does it make a difference if you zero pad the beginning or the end of the signal? Why?

No. Zero padding by n_o at the beginning of the signal introduces a time shift, multiplying the non-padded signal by $e^{-j2\pi n_o k/N}$ which has a magnitude of 1, therefore zero padding at the beginning does not change $|X[k]|$, only $\angle X[k]$.