Sampling

1. Let $y(t) = cos(6\,\pi\,t) + 2\,cos(14\,\pi\,t) - 5\,cos(26\,\pi\,t)$

   a) find the minimum sampling frequency to prevent aliasing

   max freq in y(t) is 13 Hz ⇒ must sample at a minimum of $\boxed{26\text{ Hz}}$ to prevent aliasing.

   b) find y[n] if sampled at 10Hz. Keep all discrete frequencies between 0 and π rads/sec.

   $f_s = 10\text{ Hz} \Rightarrow T_s = \frac{1}{10}$ second (ie $\frac{1}{10}$ sec between samples)

   $$\begin{aligned} y[n] &= y(nT_s) \\ &= \cos(6\pi n \tfrac{1}{10}) + 2\cos(14\pi n \tfrac{1}{10}) - 5\cos(26\pi n \tfrac{1}{10}) \\ &= \cos(0.6\pi n) + 2\cos(1.4\pi n) - 5\cos(2.6\pi n) \\ &= \cos(0.6\pi n) + 2\cos(1.4\pi n - 2\pi n) - 5\cos(2.6\pi n - 2\pi n) \\ &= \cos(0.6\pi n) + 2\cos(-0.6\pi n) - 5\cos(0.6\pi n) \\ &= \cos(0.6\pi n) + 2\cos(0.6\pi n) - 5\cos(0.6\pi n) \\ &= \boxed{-2\cos(0.6\pi n)} \end{aligned}$$

2. Let $y[n] = 2\,cos(\frac{1}{2}\pi n)$ What was original signal if f_s = 7Hz assuming no aliasing?

   $y[n] = y(t = nT_s)$. To go in reverse note $t = nT_s \Rightarrow n = \frac{t}{T_s}$

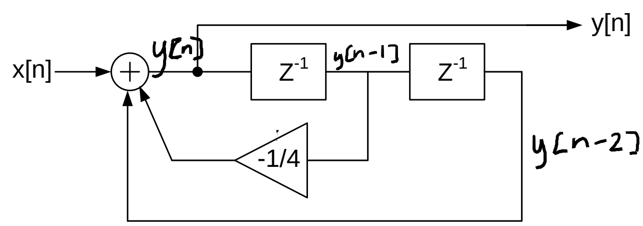
   $$\begin{aligned} y(t) &= y[n = \tfrac{t}{T_s}] \\ &= 2\cos(\tfrac{1}{2}\pi \tfrac{t}{1/7}) \\ &= \boxed{2\cos(\tfrac{7}{2}\pi t)} \end{aligned}$$

Block Diagram ↔ DE

3. a) Find DE

   1) Give signal lines a name before delays (eg $w[n]$) and then name after delay (eg $w[n-1]$)
   2) write eqn at output of each summer
   3) Put in standard form ($y[n]$ on left, $x[n]$ on right)

   1



   2 $y[n] = x[n] - \frac{1}{4}y[n-1] + y[n-2]$

   3 $\boxed{y[n] + \frac{1}{4}y[n-1] - y[n-2] = x[n]}$

b) Let $x[n] = n^2u[n]$. Plot y[n] for $0 \le n \le 2$

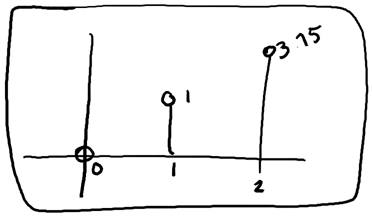
Subsitute in values of $x[n]$ into the DE and solve for the first 3 output values (i.e. $n = 0, 1, 2$)

$y[n] = -\frac{1}{4}\, y[n-1] + y[n-2] + x[n]$, $x[n] = n^2\, u[n]$

$y(0) = \frac{1}{4}(0) + (0) + 0 = 0$

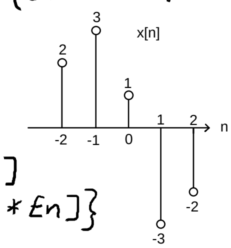
$y(1) = \frac{1}{4}(0) + (0) + 1 = 1$

$y(2) = \frac{1}{4}(1) + (0) + 4 = 3.75$



Symmetry

4. Find the ca part of x[n]



$x[n] = [2\ 3\ \underset{\uparrow}{1}\ -3\ -2]$

$x_{ca}[n] = \frac{1}{2}\{x[n] - x^*[-n]\}$

$= \frac{1}{2}\{[2\ 3\ 1\ -3\ -2] - [-2\ -3\ 1\ 3\ 2]\}$

$= \frac{1}{2}\{[4\ 6\ 0\ -6\ -4]\}$

$= \boxed{[2\ 3\ \underset{\uparrow}{0}\ -3\ -2]}$

or, Could solve geometrically by flipping $x[n]$ around horizontal axes, taking conjugate, and averaging with original.

5. Find k to make the following finite-length sequence ω[n] periodic conjugate symmetric

$\omega[n] = [\underset{\uparrow}{9}\ \ 3\text{-}j\ \ \text{-}1\ \ k]$

solving geometrically, first construct periodic extension

$\ldots\ 9\ \ 3-j\ \ -1\ \ k\ \ \boxed{\underset{\uparrow}{9}\ \ 3-j\ \ -1\ \ k}\ \ 9\ \ 3-j\ \ -1\ \ k \ldots$

for pcs, values symmetric around ↑ must be complex conjugates

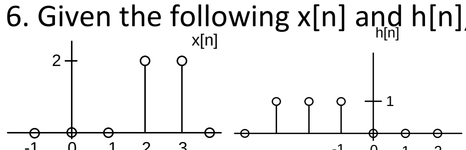
(for pca, values symmetric around ↑ must be negative complex conjugates)

$\ldots\ 9\ \ 3-j\ \ -1\ \ k\ \ \boxed{\underset{\uparrow}{9}\ \ 3-j\ \ -1\ \ k}\ \ 9\ \ 3-j\ \ -1\ \ k \ldots$

so, $k = (3-j)^*$, $\boxed{k = 3+j}$

Convolution

6. Given the following x[n] and h[n], graphically convolve to find y[n] = x[n] × h[n]



to find y[0]

x[n]

flip $h[n] \to h[-n]$

mult $x[n]\, h[-n]$

Sum: $2+2=4$

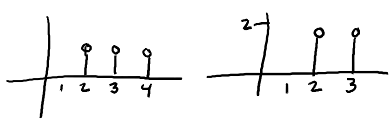
$y[0] = 4$

y[1]

shift flipped h[n] by 1 to the right

$h[1-n]$

multiply $x[n]\, h[1-n]$



Sum: $2+2=4$

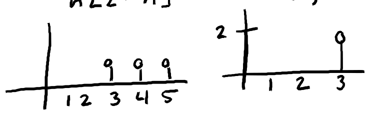
$y[1] = 4$

y[2]

Shift flipped y[n] to right by 2

$h[2-n]$

multiply $x[n]\, h[2-n]$



sum: 2

$y[2] = 2$

y[3, 4, 5, etc]

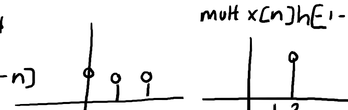
there will be no overlap of shifted $h[-n]$ with $x[n]$ so mult will be all 0's so sum will be 0.

$y[n>2] = 0$

y[-1]

shift flipped h[n] to left by 1 ⇒ $h[-1-n]$

mult $x[n]\, h[-1-n]$



Sum = 2

$y[-1] = 2$

y[-2, -3, etc]

there will be no overlap of $x[n]$ with $h[-n]$ left shifted by 2 or more

so, $y[n < -1] = 0$

y[n]