1. Consider the system $y[n] = x[n+1] – 2x[n] + x[n-1]$

   a. Is it linear?

   let $y_1[n] = x_1[n+1] – 2x_1[n] + x_1[n-1]$
   then let $x_2[n] = a\, x_1[n]$
   $y_2[n] = x_2[n+1] – 2\, x_2[n] + x_2[n-1]$
   $= a\, x_1[n+1] – 2\, a\, x_1[n] + a\, x_1[n-1]$
   $= a\{\ x_1[n+1] – 2x_1[n] + x_1[n-1]\ \}$
   $= a\, y_1\,[n]$
   $\rightarrow$ scales

   Similarly, let $x_1[n]$ cause $y_1[n]$, and $x_2[n]$ cause $y_2[n]$. Then the output $y_3[n]$ to $x_3[n] = x_1[n] + x_2[n]$ is
   $y_3[n] = \{\ x_1[n+1] + x2[n+1]\ \} – 2\ \{\ x_1[n] + x2[n]\ \} + \{\ x_1[n-1] + x2[n-1]\ \}$
   $= \{\ x_1[n+1 + 2\, x_1[n] + x_1[n-1]\ \} + \{\ x_2[n+1 + 2\, x_2[n] + x_2[n-1]\ \}$
   $= y_1[n] + y_2[n]$
   $\rightarrow$ obeys superposition

   Since both scales and obeys superposition, $\rightarrow$ **linear**

   b. Is it time-invariant?

   Yes; sample n is not an explicit argument

2. Consider a moving average filter of length 3 and the input signal $x[n] = \delta[n] + 3\delta[n-1] - 4\delta[n-2]$.

   a. Find y[0], y[1], y[2], y[3], y[4] (i.e. evaluate to 5 numbers)

   $y[0] = \{x[0] + x[-1] + x[-2]\ \} / 3 = \{1 + 0 + 0\}/3 = 1/3$. Similarly,
   y[n] = [1/3 4/3 0 -1/3 -4/3]
   ↑

   b. Find the energy in x[n]

   $1^2 + 3^2 + (-4)^2 =$ **26**

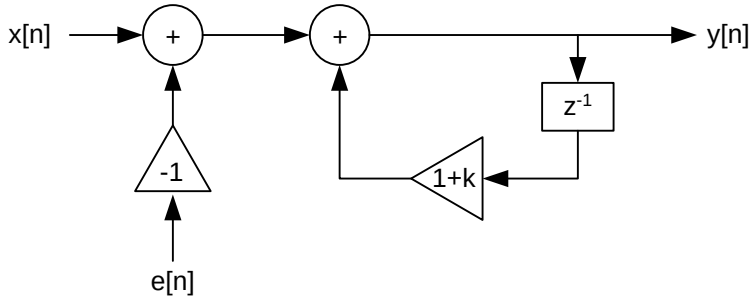
   c. Find the energy in y[n]

   Similarly to above, $1/9 + 16/9 + 1/9 + 16/9 =$, **$34/9 \approx 3.77$**

   d. Does this example suggest the system is lossless? Passive? (Note: To prove the system is lossless or passive you must prove it is so for all possible inputs – I am asking only for this specific input).

   For this example it appears **passive, but not lossless**

3. A model of your savings account y[n] at month n may look as follows



where x[n] is your monthly income
e[n] are your monthly expenses
k[n] is your monthly interest rate on your savings account (e.g. 0.01 for 1%)

a. If e[n] is 2500 u[n] then is the system linear? Causal? Shift-invariant?

Not linear (a zero input does not give a zero output (you may not have a job or income, but you still have expenses and the bank account will decline)

Causal (future values of the bank account depend only on current and past values of income and the bank account)

Shift invariant (the same rules apply whether you are analyzing your own savings or your children's savings accounts when they graduate from VMI).

b. If x[n] is 4000 u[n], e[n] = 2500 u[n], and k is 0.01 (i.e. 12% annual interest), calculate how much your savings account will hold after 10 years (i.e. y[120]). You may find writing a quick program in Matlab much faster than calculating it by hand, or you may be able to derive an explicit mathematical relationship.

Matlab:
```
N=120;
k=0.01;
x = 4000;
e = 2500;
y(1) = 4000-2500;
for i=2:N
    y(i) =  x - e + y(i-1) * (1+k);
end
```
y[120] = $345,058.03 (!)

c. Use Matlab to stem plot the impulse response for the first year given the above values for k if e[n] = 0.

```
N=12;
k=0.01;
y(1) = 1;
for i=2:N
    y(i) =  y(i-1) * (1+k);
end
stem(0:11,y)
```