

Given: continuous-time signal $x(t) = 36 \cos(12 t)$

1. What is the minimum sampling frequency to allow perfect restoration of the signal?

First, determine the maximum continuous time (CT) frequency in the signal. This is $\cos(\Omega t)$ in general, so for this signal the maximum (and only) frequency in the signal is:

$$\Omega = 12 \text{ rads/sec}$$

Recall the relationship between continuous time (CT) frequency in rads/sec Ω and CT frequency in Hz f is:

$$\Omega = 2 \pi f$$

so the maximum frequency in the signal in Hz is:

$$f_{\max} = 12/(2\pi) \text{ Hz} = 10/\pi \text{ Hz}$$

This means that in every second there will be $10/\pi$ complete periods of the CT signal.

The Sampling Theorem says to avoid aliasing the sampling frequency f_s must be twice the highest frequency in the signal, so

$$f_s = 2 (\text{max freq in signal}) = \boxed{12/\pi \text{ Hz}}$$

2. What is the discrete frequency of $x[n]$ if it is sampled from $x(t)$ at $f_s = 24/\pi$ samples/second?

$$f_s = 24/\pi \text{ samples/sec} \rightarrow T_s = \pi / 24 \text{ sec/sample}$$

$$x[n] = x(t = n T_s) = 36 \cos[12 (n T_s)] = 36 \cos\left[\frac{\pi}{2} n\right]$$

$$\omega = \frac{\pi}{2} \text{ samples/rad}$$

3. What is the discrete frequency of $x[n]$ if it is sampled from $x(t)$ at $f_s = 120/\pi$ samples/second?

$$f_s = 120/\pi \text{ samples/sec} \rightarrow T_s = \pi / 120 \text{ sec/sample}$$

$$x[n] = x(t = n T_s) = 36 \cos[12 n T_s] = 36 \cos\left[\frac{\pi}{10} n\right]$$

$$\omega = \frac{\pi}{10} \text{ samples/rad}$$

(note: same continuous time frequency, different discrete time frequency)

4. What is the discrete frequency of $x[n]$ if it is sampled from $x(t)$ at $f_s = 6/\pi$ samples/second?

$$f_s = 6/\pi \text{ samples/sec} \rightarrow T_s = \pi / 6 \text{ sec/sample}$$

$$x[n] = x(t = n T_s) = 36 \cos[12 n T_s] = 36 \cos[2\pi n]$$

can subtract or add multiples of 2π without affecting discrete frequency so

$$x[n] = 36 \cos(0 n) = 36 \quad (\text{i.e. } x[n] = [\dots 36 \ 36 \ 36 \ 36 \ 36 \ \dots])$$

$$\boxed{\omega = 0 \text{ samples/rad}}$$

(note: aliasing has occurred and it now looks like a DC unchanging signal of 36)