EE230 Laplace Transforms

Transform Pairs

	$f(t)$	$F(s)$
Unit impulse	$\delta(t)$	1
Unit step	$u(t)$	$\frac{1}{s}$
Unit ramp	$t\,u(t)$	$\frac{1}{s^2}$
n[th] integral of an impulse	$\int \cdots \int \delta(t)$	$\frac{1}{s^n}$
Power of t	$\frac{t^{n-1}}{(n-1)!}u(t)$	$\frac{1}{s^n}$
Derivative of an impulse	$\delta'(t)$	s
n[th] derivative of an impulse	$\delta^{(n)}(t)$	s^n
Exponential	$e^{-at}\,u(t)$	$\frac{1}{s+a}$
t times exponential	$te^{-at}\,u(t)$	$\frac{1}{(s+a)^2}$
t[n] times exponential	$\frac{1}{(n-1)!}t^{n-1}e^{-at}$	$\frac{1}{(s+a)^n}$
Sine	$\sin(\omega t)\,u(t)$	$\frac{\omega}{s^2+\omega^2}$
Cosine	$\cos(\omega t)\,u(t)$	$\frac{s}{s^2+\omega^2}$
Damped sine	$e^{-at}\sin(\omega t)\,u(t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
Damped cosine	$e^{-at}\cos(\omega t)\,u(t)$	$\frac{s+a}{(s+a)^2+\omega^2}$

Transform Properties

	$f(t)$	$F(s)$
Linearity	$c_1 f_1(t) + c_2 f_2(t)$	$c_1F_1(s) + c_2F_2(s)$
Differentiation	$\frac{d}{dt}f(t)$	$sF(s) - f(0^-)$
Double differentiation	$\frac{d^2}{dt^2}f(t)$	$s^2F(s) - sf(0^-) - f'(0^-)$
Integration	$\int_{0-}^{t} f(\tau)d\tau$	$\frac{1}{s}F(s)$
Time shift	$f(t-t_0)u(t-t_0), \quad t_0 > 0$	$e^{-st_0}F(s)$
Convolution	$f_1(t) * f_2(t)$	$F_1(s)F_2(s)$
Initial value	$f(0)$	$\lim_{s\to\infty} sF(s)$
Final value	$f(\infty)$	$\lim_{s\to 0} sF(s)$