Useful Equations for Fourier Series Problems

Integrals

$$\int \cos(\omega t)\,dt = \frac{1}{\omega}\sin(\omega t)$$

$$\int \sin(\omega t)\,dt = -\frac{1}{\omega}\cos(\omega t)$$

$$\int t\cos(\omega t)\,dt = \frac{1}{\omega^2}\cos(\omega t) + \frac{t}{\omega}\sin(\omega t)$$

$$\int t\sin(\omega t)\,dt = \frac{1}{\omega^2}\sin(\omega t) - \frac{t}{\omega}\cos(\omega t)$$

Your Own

Trig Products

$$\sin(a)\cos(b) = \tfrac{1}{2}[\sin(a+b) + \sin(a-b)]$$

$$\cos(a)\cos(b) = \tfrac{1}{2}[\cos(a+b) + \cos(a-b)]$$

$$\sin(a)\sin(b) = \tfrac{1}{2}[\cos(a-b) - \cos(a+b)]$$

Euler's Identities

$$\cos(\omega) = \tfrac{1}{2}[e^{j\omega} + e^{-j\omega}]$$

$$\sin(\omega) = \tfrac{1}{j2}[e^{j\omega} - e^{-j\omega}]$$

$$e^{j\omega} = \cos(\omega) + j\sin(\omega)$$

Fourier Series

given *f(t)* periodic in T so $\omega_o = 2\pi / T$

$$f(t) = a_o + \sum_{n=1}^{\infty} a_n \cos(n\omega_o t) + b_n \sin(n\omega_o t) = A_o + \sum_{n=1}^{\infty} A_n \cos(n\omega_o t + \phi_n) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_o t}$$

coefficient formulae

$$a_o = \frac{1}{T}\int_{<T>} f(t)\,dt, \quad a_n = \frac{2}{T}\int_{<T>} f(t)\cos(n\omega_o t)\,dt, \quad b_n = \frac{2}{T}\int_{<T>} f(t)\sin(n\omega_o t)\,dt$$

coefficient relationships

$$A_o = a_o = c_o, \qquad A_n \angle \phi_n = a_n - jb_n = 2c_n$$

Your Own