EE230 Final Exam Review

Questions

1. Laplace Transform
   — Forward and inverse
   — Using integral definition and tables/properties
2. Circuits and Laplace Transforms
   — With/without initial conditions
   — Convert among forms - H(s), h(t), differential equations, pole/zeros, Bode Plot
3. Filter Design
   — Real-world problem into specifications (e.g. 4^{th} order Butterworth LP, cutoff 12Hz)
4. Bode Plots
   — With and without poles and/or zeros at the origin
   — Magnitude aka Amplitude aka Gain
   — H(s) to Bode Plot and Bode Plot to H(s)
5. Fourier Series decomposition of a periodic function
   — cos/sin
   — cos/ϕ (i.e. Spectral Magnitude, or Spectral Phase plots)
   — complex exponential
6. Fourier Transform of a function
   — Forward and inverse
   — Using integral definition and tables/properties

Notes

- Nervous? You should be a little before a major exam! Overcome nervousness with adequate preparation so you can walk in confidently.
- Study by solving problems that cover the questions.
  - The above questions are specific and cover the skills you need for DSP, Controls, and Communications next year.
  - Look at Tests 1,2 to find areas of weakness and solve problems from
    - Textbook examples and solved problems
    - Old homeworks
    - Collaborative problems
    - Worked problems in the extra textbooks I have left in class
- To limit the length of the exam, only 5 of the 6 questions above will appear.
- I will provide the three attached equation sheets you have previously seen for
  - Laplace Transforms
  - Fourier Series
  - Fourier Transforms
- You may bring a
  - a hand calculator,
  - three 3x5 cards, both sides, of your notes. No restrictions as long as not copied.
  - A clean FE handbook that you personally own.

Laplace Transforms

Transform Pairs

	$f(t)$	$F(s)$
Unit impulse	$\delta(t)$	1
Unit step	$u(t)$	$\frac{1}{s}$
Unit ramp	$t\,u(t)$	$\frac{1}{s^2}$
n^{th} integral of an impulse	$\int \cdots \int \delta(t)$	$\frac{1}{s^n}$
Power of t	$\frac{t^{n-1}}{(n-1)!}u(t)$	$\frac{1}{s^n}$
Derivative of an impulse	$\delta'(t)$	s
n^{th} derivative of an impulse	$\delta^{(n)}(t)$	s^n
Exponential	$e^{-at}\,u(t)$	$\frac{1}{s+a}$
t times exponential	$te^{-at}\,u(t)$	$\frac{1}{(s+a)^2}$
t^n times exponential	$\frac{1}{(n-1)!}t^{n-1}e^{-at}$	$\frac{1}{(s+a)^n}$
Sine	$\sin(\omega t)u(t)$	$\frac{\omega}{s^2+\omega^2}$
Cosine	$\cos(\omega t)u(t)$	$\frac{s}{s^2+\omega^2}$
Damped sine	$e^{-at}\sin(\omega t)u(t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
Damped cosine	$e^{-at}\cos(\omega t)u(t)$	$\frac{s+a}{(s+a)^2+\omega^2}$

Transform Properties

	$f(t)$	$F(s)$
Linearity	$c_1 f_1(t) + c_2 f_2(t)$	$c_1F_1(s) + c_2F_2(s)$
Differentiation	$\frac{d}{dt}f(t)$	$sF(s) - f(0^-)$
Double differentiation	$\frac{d^2}{dt^2}f(t)$	$s^2F(s) - s\,f(0^-) - f'(0^-)$
Integration	$\int_{0-}^{t} f(\tau)d\tau$	$\frac{1}{s}F(s)$
Time shift	$f(t-t_0)u(t-t_0), \quad t_0 > 0$	$e^{-st_0}F(s)$
Convolution	$f_1(t) * f_2(t)$	$F_1(s)F_2(s)$
Initial value	$f(0)$	$\lim_{s\to\infty} sF(s)$
Final value	$f(\infty)$	$\lim_{s\to 0} sF(s)$

Fourier Series

Integrals

$$\int \cos(\omega t)dt = \frac{1}{\omega}\sin(\omega t)$$

$$\int \sin(\omega t)dt = -\frac{1}{\omega}\cos(\omega t)$$

$$\int t\cos(\omega t)dt = \frac{1}{\omega^2}\cos(\omega t) + \frac{t}{\omega}\sin(\omega t)$$

$$\int t\sin(\omega t)dt = \frac{1}{\omega^2}\sin(\omega t) - \frac{t}{\omega}\cos(\omega t)$$

Your Own

Trig Products

$$\sin(a)\cos(b) = \tfrac{1}{2}[\sin(a+b)+\sin(a-b)]$$

$$\cos(a)\cos(b) = \tfrac{1}{2}[\cos(a+b)+\cos(a-b)]$$

$$\sin(a)\sin(b) = \tfrac{1}{2}[\cos(a-b)-\cos(a+b)]$$

Euler's Identities

$$\cos(\omega) = \tfrac{1}{2}[e^{j\omega}+e^{-j\omega}]$$

$$\sin(\omega) = \tfrac{1}{j2}[e^{j\omega}-e^{-j\omega}]$$

$$e^{j\omega} = \cos(\omega) + j\sin(\omega)$$

Fourier Series

given $f(t)$ periodic in T so $\omega_o = 2\pi / T$

$$f(t) = a_o + \sum_{n=1}^{\infty} a_n\cos(n\omega_o t) + b_n\sin(n\omega_o t) = A_o + \sum_{n=1}^{\infty} A_n\cos(n\omega_o t + \phi_n) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_o t}$$

coefficient formulae

$$a_o = \frac{1}{T}\int_{<T>} f(t)dt, \quad a_n = \frac{2}{T}\int_{<T>} f(t)\cos(n\omega_o t)dt, \quad b_n = \frac{2}{T}\int_{<T>} f(t)\sin(n\omega_o t)dt$$

coefficient relationships

$$A_o = a_o = c_o, \qquad A_n\angle\phi_n = a_n - jb_n = 2c_n$$

Your Own

EE230 Fourier Transforms

Transform Pairs

	$f(t)$	$F(\omega)$
Definition	$f(t)$	$\int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$
Unit impulse	$\delta(t)$	1
Unity	1	$2\pi\delta(\omega)$
Unit step	$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
Pulse, centered ± τ wide	$u(t+\tau) - u(t-\tau)$	$\frac{2\sin(\omega\tau)}{\omega}$ (aka "sinc")
Unit ramp (V-shaped)	$\lvert t \rvert$	$\frac{-2}{\omega^2}$
Exponential, right sided	$e^{-at}\, u(t)$	$\frac{1}{a + j\omega}$
Exponential, left sided	$e^{at}\, u(-t)$	$\frac{1}{a - j\omega}$
t^n times exponential	$t^n e^{-at} u(t)$	$\frac{n!}{(a + j\omega)^{n+1}}$
Exponential, both-sided	$e^{-a\lvert t\rvert}$	$\frac{2a}{a^2 + \omega^2}$
Complex exponential	$e^{j\omega_o t}$	$2\pi\delta(\omega - \omega_o)$
Sine	$\sin(\omega_o t)$	$j\pi[\delta(\omega + \omega_o) - \delta(\omega - \omega_o)]$
Cosine	$\cos(\omega_o t)$	$\pi[\delta(\omega + \omega_o) + \delta(\omega - \omega_o)]$
Damped sine	$e^{-at} \sin(\omega_o t) u(t)$	$\frac{\omega_o}{(a + j\omega)^2 + \omega_o^2}$
Damped cosine	$e^{-at} \cos(\omega_o t) u(t)$	$\frac{a + j\omega}{(a + j\omega)^2 + \omega_o^2}$

Transform Properties

	$f(t)$	$F(\omega)$
Linearity	$c_1 f_1(t) + c_2 f_2(t)$	$c_1 F_1(\omega) + c_2 F_2(\omega)$
Time shift	$f(t - t_0)$	$e^{-j\omega t_0} F(\omega)$
Frequency shift	$e^{j\omega_o t} f(t)$	$F(\omega - \omega_o)$
Time differentiation	$\frac{d}{dt} f(t)$ $\frac{d^n}{dt^n} f(t)$	$j\omega F(\omega)$ $(j\omega)^n F(\omega)$
Time integration	$\int_{-\infty}^{t} f(\tau)\, d\tau$	$\frac{F(\omega)}{j\omega} + \pi F(0)\, \delta(\omega)$
Convolution in t	$f_1(t) * f_2(t)$	$F_1(\omega) F_2(\omega)$
Convolution in ω	$f_1(t)\, f_2(t)$	$\frac{1}{2\pi} F_1(\omega) * F_2(\omega)$