

**Given:**  $f(t) = 2t$  for  $-1 \leq t < 1$ , and repeating outside those limits

**Find:** The middle 3 complex Fourier Series coefficients  $c_{-1}$ ,  $c_0$ , and  $c_1$

**Hint:**  $\int t e^{at} dt = \frac{t}{a} e^{at} - \frac{1}{a^2} e^{at}$

Looks like a sawtooth wave form:

$c_0 = 0$  by inspection

$$c_1 = \frac{1}{T} \int_{\langle T \rangle} f(t) e^{-j\omega_0 t} dt, \quad T = 2 \Rightarrow \omega_0 = \frac{2\pi}{T} = \pi$$

$$= \frac{1}{2} \int_{-1}^1 2t e^{-j\pi t} dt$$

$$= \int_{-1}^1 t e^{-j\pi t} dt$$

$$= \left[ \frac{t}{-j\pi} e^{-j\pi t} - \frac{1}{(j\pi)^2} e^{-j\pi t} \right]_{t=-1}^1$$

$$= \frac{j}{\pi} \left[ t e^{-j\pi t} \right]_{t=-1}^1 + \frac{1}{\pi^2} \left[ e^{-j\pi t} \right]_{t=-1}^1$$

$$= \frac{j}{\pi} \left[ e^{-j\pi} - (-1) e^{j\pi} \right] + \frac{1}{\pi^2} \left[ e^{-j\pi} - e^{j\pi} \right]$$

$$= \frac{-j^2}{\pi} + \frac{1}{\pi^2} \cdot 0$$

$$= \boxed{-j0.637 = 0.637 \angle -90^\circ}$$

$$c_{-1} = c_1^* = \boxed{j0.637 = 0.637 \angle 90^\circ}$$