

All questions refer to a power generator producing a voltage of

$$v(t) = 30 + 20 \cos(60\pi t + 45^\circ) + 10 \cos(120\pi t + 45^\circ) \text{ V}$$

with an exiting current of

$$i(t) = 6 + 4 \cos(60\pi t + 10^\circ) + 2 \cos(120\pi t + 120^\circ) \text{ A}$$

**Find:** The  $V_{\text{RMS}}$  of the voltage waveform

$$\begin{aligned} V_{\text{RMS}}^2 &= A_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} A_n^2 \\ &= 30^2 + \frac{1}{2} (20^2 + 10^2) \\ &= 900 + \frac{1}{2} (500) \\ &= 1150 \\ V_{\text{RMS}} &= \sqrt{1150} = \boxed{33.9 \text{ V}_{\text{RMS}}} \end{aligned}$$

**Find:** The  $I_{\text{RMS}}$  of the voltage waveform

$$\begin{aligned} I_{\text{RMS}}^2 &= 6^2 + \frac{1}{2} (4^2 + 2^2) \\ &= 46 \\ I_{\text{RMS}} &= \boxed{6.78 \text{ A}_{\text{RMS}}} \end{aligned}$$

**Find:** The total average power delivered by the generator

$$\begin{aligned} P_{\text{TOTAL}} &= V_{\text{DC}} I_{\text{DC}} + \frac{1}{2} \sum_{n=1}^{\infty} V_n I_n \cos(\phi_{V_n} - \phi_{I_n}) \\ &= (30)(6) + \frac{1}{2} [20 \cdot 4 \cos(35^\circ) + 10 \cdot 2 \cos(-75^\circ)] \\ &= 180 + 40 \cos(35^\circ) + 10 \cos(75^\circ) \\ &= 180 + 32.8 + 2.59 \\ &= \boxed{215 \text{ W}} \end{aligned}$$

**Find:** (Real-world) The device is supposed to deliver a pure DC source but it appears to have some contamination from nearby electronic devices. What is the percentage of the total output power contaminated by 60Hz?

$$60 \text{ Hz} \rightarrow \omega = 2\pi f = 120\pi$$

$$\begin{aligned} P_{60} &= \frac{1}{2} V_{60} I_{60} \cos(\phi_{V_{60}} - \phi_{I_{60}}) \\ &= \frac{1}{2} \cdot 10 \cdot 2 \cos(75^\circ) \\ &= 2.59 \end{aligned}$$

$$\frac{P_{60}}{P_{\text{TOTAL}}} \times 100\% = \frac{2.59}{215} \cdot 100\% = \boxed{1.20\%} \text{ 60Hz power contamination}$$