Objectives

- Instantaneous Power
- Average Power
- Parseval's Theorem
    - Example of Parseval's
- Real-World Example

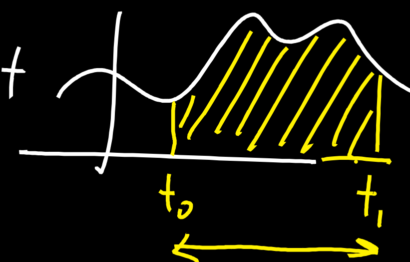
Instantaneous Power

$$\boxed{p(t) = i(t) \cdot v(t)}$$ any waveform, periodic or not

Average Power (only periodic)

$$P = \frac{1}{T} \int_{\langle T \rangle} p(t)\, dt$$ any periodic waveform

$$P = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} p(t)\, dt$$



$$= \frac{1}{2} V_M I_M \underline{\cos(\theta_v - \theta_I)}$$ for SSS = $\begin{cases} v(t) = V_M \cos(\omega t + \theta_v) \\ i(t) = I_M \cos(\omega t + \theta_I) \end{cases}$

$$= V_{DC} \cdot I_{DC} + \frac{1}{2} \sum_{n=1}^{\infty} V_n I_n \cos(\phi_{vn} - \phi_{In})$$ for any periodic $\begin{cases} v(t) = V_{DC} + \sum_{n=1}^{\infty} V_n \cos(n\omega_0 t + \phi_{vn}) \\ i(t) = I_{DC} + \sum_{n=1}^{\infty} I_n \cos(n\omega_0 t + \phi_{in}) \end{cases}$

Power in a harmonic (if a voltage livered to a $1\,\Omega$ resistor) $\begin{cases} A_0^2 & \text{if } n = 0 \\ \frac{1}{2} A_n^2 & \text{if } n > 0 \end{cases}$

Parseval's Theorem

Average power P of periodic signal $\underline{v(t)}$ = DC average power + AC average powers

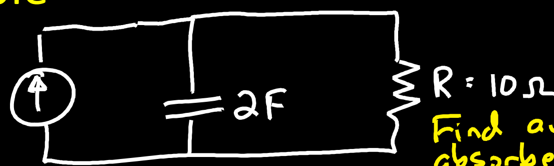
$$P = V_{Rms}^2 = \underline{\frac{1}{T}\int_{\langle T\rangle} v^2(t)\,dt} = \begin{cases} A_0^2 + \frac{1}{2}\sum_{n=1}^{\infty} A_n^2 \\ \underline{a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty} (a_n^2 + b_n^2)} \end{cases}$$

So? ① Lets you compute V_{Rms} quickly given FS

② " " determine what % total power is at a specific frequency

Parseval's Example

$2 + 10\cos(t + 10^\circ) + 6\cos(3t + 55^\circ)$ A



Find average power absorbed by R

$$\boxed{P = V_{rms}^2 = \frac{1}{T}\int_{<T>} v^2(t)\,dt = \begin{cases} A_0^2 + \frac{1}{2}\sum_{n=1}^{\infty} A_n^2 \\ a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty} (a_n^2 + b_n^2) \end{cases}}$$

$$P_{ave} = P_{DC} + P_{freq1} + P_{freq2} + \dots$$

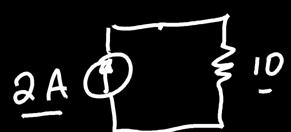
$$= \underline{V_{DC}\, I_{DC}} + \underline{\frac{1}{2}\sum_{n=1}^{\infty} (V_n \cdot i_n)\cos(\phi_{vn} - \phi_{in})}$$

$$\frac{1}{j \cdot 1 \cdot 2} = \frac{1}{j2} = -j\frac{1}{2}$$

$$\frac{1}{j} \cdot \frac{j}{j} = \frac{j}{j^2} = \frac{j}{-1} = -j$$

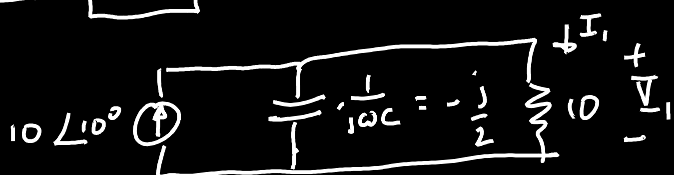
$$Z = \left(-\frac{j}{2}\right) // 10 = \frac{-j5}{10 - j/2} = \frac{-j10}{20 - j}$$

① DC



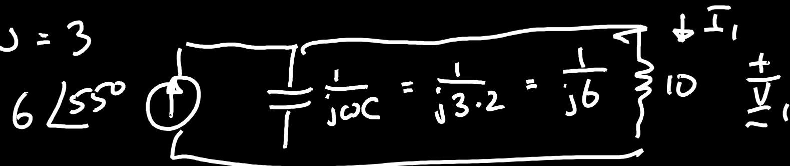
$$\left.\begin{aligned} V_{DC} &= 20V \\ I_{DC} &= 2A \end{aligned}\right\} P_{DC} = 40W$$

② $\omega = 1$



$$\left.\begin{aligned} V_1 &= (10\angle 10^\circ)\left(\frac{-j10}{20 - j}\right) = 5\angle -77.1^\circ \\ I_1 &= \frac{5\angle -77.1}{10} = 0.5\angle -77.1^\circ \end{aligned}\right\} P_1 = \frac{1}{2}(5)(0.5)\,\cancel{\cos(0^\circ)}^{\,1} = 1.25W$$

③ $\omega = 3$



$$\left.\begin{aligned} V_3 &= (6\angle 55^\circ)\left(\frac{1}{j6} // 10\right) = 1\angle -54^\circ \\ I_3 &= V_3/10 = 0.1\angle -54^\circ \end{aligned}\right\} P_3 = \frac{1}{2}(1)(0.1)\cos(0) = 0.05$$

$$P_{ave} = 40 + 1.25 + 0.05 = \boxed{41.3W}$$

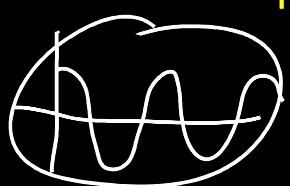
Parseval's Example 2

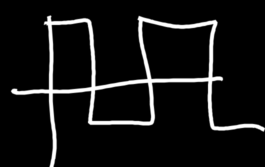
$v(t) = 3 + 2\cos(2t) - \cos(4t - 20^\circ)$ Find V_{RMS}

Method 1: $V_{RMS} = \sqrt{\frac{1}{T}\int_{\langle T \rangle} v^2(t)\,dt}$ Painful!

Method 2:
$$P = V_{RMS}^2 = A_0^2 + \frac{1}{2}\sum_{n=1}^{\infty} A_n^2$$
$$= 3^2 + \frac{1}{2}\left(2^2 + (-1)^2\right)$$
$$= 9 + \frac{1}{2}(5)$$
$$= 11.5$$
$$V_{RMS} = \sqrt{11.5} = \boxed{3.39\ V_{RMS}}$$

Real World Example





99% spectrally pure

$$\text{Test} = \frac{\text{Power } 60\text{ Hz}}{\text{Total Power}} \geq 0.99$$

$$\boxed{\frac{\frac{1}{2}A_1^2}{V_{RMS}^2} \geq 0.99}$$