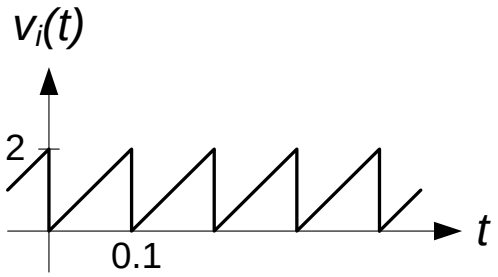
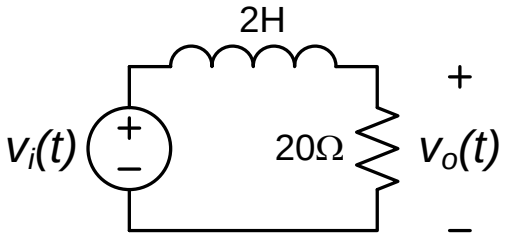




Given: The above periodic input to the above circuit fragment
Find: An approximation to $v_o(t)$ consisting of the DC and first two harmonics. Coefficients should be numeric, not functional (e.g. 1.27 not $4/\pi$).

① Find a,b coeffs. of $v_i(t)$.

- No symmetry to help
- $a_0 = 1$ by inspection (above/below one equally)
- $a_n = \frac{2}{T}\int_{<T>} f(t)\cos(n\omega_0 t)\,dt$. $T = 0.1 \Rightarrow \omega_0 = 20\pi$. $f(t) = \frac{2}{.1}t = 20t$ $0<t<0.1$

$$= 20\int_0^{0.1} 20t\cos(n20\pi t)\,dt$$

$$= 400\left[\left(\frac{1}{n20\pi}\right)^2\cos(n20\pi t) + \frac{t}{20\pi}\sin(n20\pi t)\right]_{t=0}^{0.1}$$

$$= 400\left(\frac{1}{n20\pi}\right)^2\left[\cos(n2\pi) - \cos(0)\right] + \frac{400}{20\pi}\left[0.1\sin(n2\pi) - 0\right]$$

$= 0$ ⇐ in retrospect, "obvious". Subtract 1 from waveform and it becomes odd ⇒ $a_n = 0$ except for the DC value of 1.

- $b_n = \frac{2}{T}\int_{<T>} f(t)\sin(n\omega_0 t)\,dt$

$$= 20\int_0^{0.1} 20t\sin(n20\pi t)\,dt$$

$$= 400\left[\left(\frac{1}{n20\pi}\right)^2\sin(n20\pi t) - \frac{t}{n20\pi}\cos(n20\pi t)\right]_{t=0}^{0.1}$$

$$= 400\left(\frac{1}{n20\pi}\right)^2\left[\sin(n2\pi) - \sin(0)\right] - \frac{400}{n20\pi}\left[0.1\cos(n2\pi) - 0\right]$$

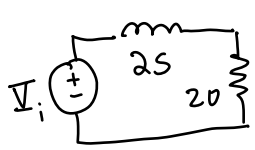
$$= -\frac{20}{n\pi}(0.1) = -\frac{2}{n\pi}$$

② Find A_n, ϕ_n coeffs of input

$A_0 = a_0 = 1$

$A_n\angle\phi_n = a_n - jb_n = \frac{j2}{n\pi} = \frac{2}{n\pi}\angle 90°$

③ Find $H(\omega)$



$$V_o = V_i \frac{20}{2s+20} \Rightarrow H(s) = \frac{10}{s+10} \Rightarrow H(\omega) = \frac{10}{j\omega+10}$$

④ Fill out table

n	$\omega = \omega_0 n$	$A_n \angle \Phi_n$	$H(\omega) = \frac{10}{j\omega+10}$	$A_n' \angle \phi' = (A_n \angle \phi_n) H(\omega)$
0	0	$1 \angle 0^\circ$	$H(0) = 1$	1
1	20π	$\frac{2}{n\pi} \angle 90^\circ = 0.637 \angle 90^\circ$	$H(20\pi) = 0.157 \angle -81^\circ$	$0.1 \angle 9^\circ$
2	40π	$\frac{2}{2n\pi} \angle 90^\circ = 0.318 \angle 90^\circ$	$H(40\pi) = 0.079 \angle -85^\circ$	$0.025 \angle 5^\circ$

⑤ Reconstruct $v(t)$

$$v(t) = 1 + 0.1\cos(20\pi t + 9^\circ) + 0.025\cos(40\pi t + 5^\circ) \text{ V}$$