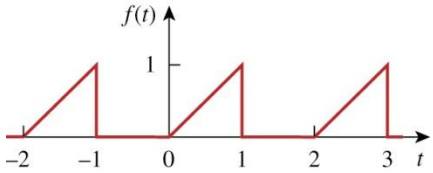
All problems use the following periodic waveform $g(t)$. **For all problems give numeric answers**, e.g. not $2/\pi$ but 0.6366.



P1 **Find:** T, f_o, ω_o.

$T = 2$, $f_o = \frac{1}{T} = 0.5$, $\omega_o = 2\pi f_o = 3.14$

P2 **Find:** The trig Fourier Series coefficients a_0, a_1, a_2, b_1, b_2 for $f(t)$ above, so $a_0+a_1\cos(\omega_o t)+b_1\sin(\omega_o t)+a_2\cos(2\omega_o t) + b_2\sin(2\omega_o t)$ approximates $f(t)$.
The answers must be *numeric*, e.g. not $2/\pi$ but 0.6366.

note: $f(t) = \begin{cases} t, & 0 \le t < 1 \\ 0, & 1 \le t < 2 \end{cases}$

$$a_0 = \frac{1}{T}\int_{<T>} f(t)\,dt = \frac{1}{2}\left[\int_0^1 t\,dt + \int_1^2 0\,dt\right] = \frac{1}{4}t^2\Big|_{t=0}^{1} = \frac{1}{4}$$

$$a_n = \frac{2}{T}\int_{<T>} f(t)\cos(n\omega_o t)\,dt = \int_0^1 t\cos(n\pi t)\,dt \quad \text{use eqn sheet}$$

$$= \frac{1}{(n\pi)^2}\cos(n\pi t) + \frac{t}{n\pi}\sin(n\pi t)\Big]_{t=0}^{1}$$

$$= \frac{1}{(n\pi)^2}\left[\cos(n\pi) - \cos(0)\right] + \left[\frac{1}{n\pi}\sin(n\pi) - 0\right]$$

$$= \frac{1}{(n\pi)^2}\left[(-1)^n - 1\right]$$

$$= \begin{cases} \frac{-2}{(n\pi)^2} & n \text{ odd} \\ 0 & \text{otherwise} \end{cases} \quad \text{so} \quad \boxed{a_1 = \frac{-2}{\pi^2} = -0.203,\ a_2 = 0}$$

$$b_n = \frac{2}{T}\int_{<T>} f(t)\sin(n\omega_o t)\,dt = \int_0^1 t\sin(n\pi t)\,dt$$

$$= \frac{1}{(n\pi)^2}\sin(n\pi t) - \frac{t}{n\pi}\cos(n\pi t)\Big]_{t=0}^{1}$$

$$= \frac{1}{(n\pi)^2}\left[\sin(n\pi t) - \sin(0)\right] - \left[\frac{1}{n\pi}\cos(n\pi) - 0\right]$$

$$= -\frac{1}{n\pi}(-1)^n$$

$$= \begin{cases} -\frac{1}{n\pi}, & n \text{ even} \\ \frac{1}{n\pi}, & n \text{ odd} \end{cases} \quad \text{so} \quad \boxed{b_1 = \frac{1}{\pi} = 0.318,\ b_2 = -\frac{1}{2\pi} = -0.159}$$

P3 Find: The coefficients A_0, A_1, A_2, ϕ_1, ϕ_2 so that $A_0 + A_1 \cos(\omega_o t + \phi_1) + A_2 \cos(2\omega_o t + \phi_2)$ approximates $f(t)$. Keep the answers must be numeric, e.g. not $2/\pi$ but 0.6366.

$A_0 = a_0 = 0.25$

$C_1 = a_1 - jb_1 = -0.203 + j0.318 = 0.377 \angle 123^\circ \Rightarrow A_1 = 0.377, \phi_1 = 123^\circ$

$C_2 = a_2 - jb_2 = -j0.159 = 0.159 \angle -90^\circ \Rightarrow A_2 = 0.159, \phi_2 = -90^\circ$

P4 Find: Sketch the amplitude spectra and phase spectra of g(t) for $0 \le \omega \le 2\omega_o$. Remember to label the ω axis with numbers (e.g. not $2\omega_o$ but 2.52).

```
>> A=[0.25 0.377 0.159];
>> w=[0:2]*pi;
>> subplot(1,2,1);
>> stem(w,A,'k', 'linewidth',2)
>> title('Amplitude spectra'),xlabel('freq (rad/sec)')
>> phi=[123 -90];
>> w=[1:2]*pi;
>> subplot(1,2,2);
>> stem(w,phi,'k', 'linewidth',2)
>> title('Phase spectra'),xlabel('freq (rad/sec)')
```