1. $v(t) = 3te^{-2t}u(t-1)$ find V(s)

$3te^{-2t}u(t) \Leftrightarrow \frac{3}{(s+2)^2}$ this is the "core" of what is given

$3(t-1)e^{-2(t-1)}u(t-1) \Leftrightarrow \frac{3}{(s+2)^2}e^{-s}$ delay by 1 gives closer to what is given

$= 3te^{-2(t-1)}u(t-1) - 3e^{-2(t-1)}u(t-1) \Leftrightarrow$ same as above; distributed the $(t-1)$

$= 3te^{-2t}\cdot e^{2}u(t-1) -$ " $\Leftrightarrow$ same as above; multiplied out e^{-2t+2}

$e^{-2}\cdot$(entire above eqn) $+ 3e^{-2(t-1)}u(t-1) \Leftrightarrow \boxed{e^{-2}\cdot\frac{3}{(s+2)^2}e^{-s} + \frac{3}{s+2}e^{-s}}$ finally same as what is given

$= 3te^{-2t}u(t-1)$

2. Find $y(0)$ and $y(\infty)$ if the system is $H(s) = \frac{2}{s^2+6s+10}$ and $x(t) = 5e^{-2t}$

$= \frac{5}{s+2}\cdot\frac{2}{s^2+6s+10}$

$y(t=0) = \lim_{s\to\infty} sY(s)$

$= \lim_{s\to\infty} \frac{10s}{(s+2)(s^2+6s+10)} \div \frac{s^3}{s^3}$

$= \lim_{s\to\infty} \frac{10/s^2}{(1+2/s)(1+6/s+10/s^2)}$

$= \frac{0}{1\cdot 1}$

$= \boxed{0}$

$y(\infty) = \lim_{s\to 0} sY(s)$

$= \frac{0}{2\cdot 10}$

$= \boxed{0}$

3. Same as above but find $y(t)$.

$Y(s) = X(s)H(s)$

$= \frac{5}{s+2}\cdot\frac{2}{s^2+6s+10}$

roots: $s = \frac{-6\pm\sqrt{36-40}}{2} = -3\pm j$

$= \frac{10}{(s+2)(s+3-j)(s+3+j)}$

use the $s^2+6s+10$ form since same as $(s+3-j)(s+3+j)$

$A = \frac{10}{4-12+10} = 5$

$B = \frac{10}{((-3+j)+2)((-3+j)+3+j)} = 3.53\angle 135^\circ$

$= \frac{A}{s+2} + \frac{B}{s+3-j} + \frac{B^*}{s+3+j}$

$\boxed{y(t) = [5e^{-2t} + 7.06e^{-3t}\cos(t+135^\circ)]u(t)}$

4. Find y(t) = x(t) * h(t) by both integral and graphical methods if x(t) = -h(t) = u(t-1). (Note: on a test, the graphical problem would be limited to flat-topped functions (steps and pulses), but it would probably have exponentials if the problem specified use of integral methods).

a) integral: $x(t) = u(t-1)$

$h(t) = -u(t-1)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} u(\tau-1)(-u(t-\tau-1))\, d\tau$$

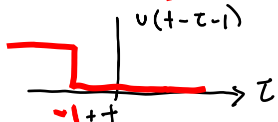
$$= -\int_{-\infty}^{\infty} u(\tau-1)\, u(t-\tau-1)\, d\tau$$

draw $u(\tau-1)\,u(t-\tau-1)$

$u(\tau-1)$

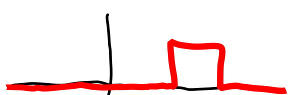
$u(t-\tau-1)$

draw for t=0 first
then add t to
switch points



$-1+t$

$u(\tau-1)\,u(t-\tau-1)$



1 $\quad -1+t$

for $-1+t > 1$
or 0 otherwise

so $u(\tau-1)\,u(t-\tau-1) =$

$$\begin{cases} \text{pulse b/n } 1 < \tau < t-1 & \text{if } t > 2 \\ 0 & \text{otherwise} \end{cases}$$

$$y(t) = \left[-\int_{1}^{t-1} 1\, d\tau\right] u(t-2)$$

$$= -\tau\Big]_{\tau=1}^{t-1}\, u(t-2)$$

$$= -(t-1-1)\,u(t-2)$$

$$= \boxed{(2-t)\,u(t-2)}$$

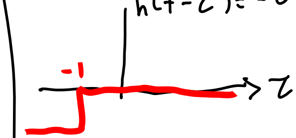
b) Graphical $\quad x(t) = u(t-1)$

$h(t) = -u(t-1)$

so $h(t-\tau) = -u(t-\tau-1)$

$x(\tau)$

$h(t-\tau) = -u(t-\tau-1)$ for $t=0$

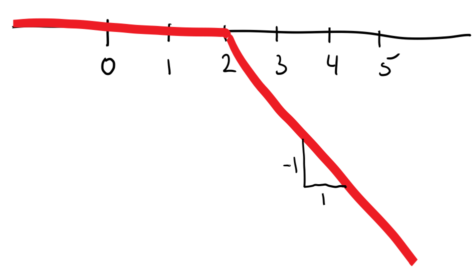


by inspection, would have to slide lower plot by $t \geq 2$ to have any interaction. Then the product would by a pulse -1 unit high from $\tau = 1$ to $\tau = -1+t$ for an area of $(-1)[(-1+t)-1] = 2-t$

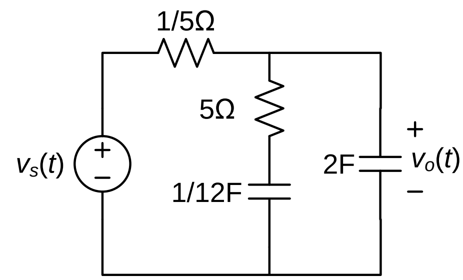
so

$$\boxed{y(t) = (2-t)\,u(t-2)}$$

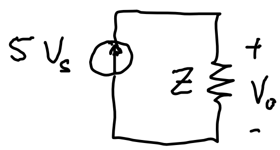
OR



5. Find the circuit
   a. transfer function
   b. impulse response
   c. differential equation
   d. s-plane plot



Could do nodal (1 eqn) or
Z simplification - about same difficulty.



$$Z = \tfrac{1}{5} \,//\, \left(5 + \tfrac{12}{s}\right) \,//\, \tfrac{1}{2s} = \frac{1}{5 + \frac{1}{\frac{5s+12}{s}} + 2s}$$

$$= \frac{1}{5 + \frac{s}{5s+12} + 2s} = \frac{5s+12}{(25s+60) + s + (10s^2+24s)}$$

$$= \frac{5s+12}{10s^2+50s+60} = \frac{\frac{1}{2}s + 6/5}{s^2+5s+6}$$

$$V_o = 5V_s \cdot Z$$

a) $H(s) = \dfrac{V_o}{V_s} = 5Z = \boxed{\dfrac{\frac{5}{2}s+6}{s^2+5s+6}}$

b) $H(s) = \dfrac{\frac{5}{2}s+6}{(s+2)(s+3)} = \dfrac{A}{s+2} + \dfrac{B}{s+3}$, $A \to \dfrac{-5+6}{1} = 1$, $B \to \dfrac{-\frac{15}{2}+6}{-1} = 1.5$

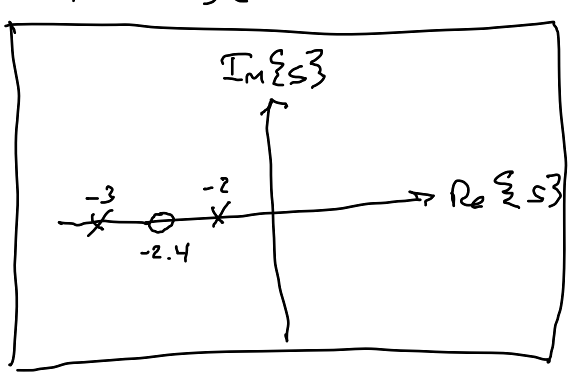
$$\boxed{h(t) = (e^{-2t} + 1.5e^{-3t})\,u(t)}$$

c) $\dfrac{V_o}{V_s} = \dfrac{\frac{5}{2}s+6}{s^2+5s+6} \Rightarrow V_o\left[s^2+5s+6\right] = V_s\left[\frac{5}{2}s+6\right]$

$$\boxed{v_o'' + 5v_o' + 6v_o = \tfrac{5}{2}v_s' + 6v_s}$$

d) zeros: $\frac{5}{2}s + 6 = 0 \Rightarrow s = \frac{2}{5}(-6) = -\frac{12}{5} = -2.4$

poles: $s = -2, -3$



6. The differential equation describing a system is $3y'(t) + y(t) = x''(t)$. Find the output $y(t)$ if the input is $x(t) = 4e^{-t/3}$. Hint: is it proper?

$$Y(s) = H(s)X(s)$$
$$= \frac{s^2}{3s+1} \cdot \frac{4}{s+1/3}$$
$$= \frac{\frac{4}{3}s^2}{(s+1/3)(s+1/3)} \quad \text{improper fraction!} = \frac{\frac{4}{3}s^2}{s^2+\frac{2}{3}s+\frac{1}{9}}$$

$$s^2+\tfrac{2}{3}s+\tfrac{1}{9} \;\big)\; \tfrac{4}{3}s^2 \quad = \tfrac{4}{3} - \frac{\frac{8}{9}s+\frac{4}{27}}{(s+\frac{1}{3})^2}$$
$$\tfrac{4}{3}s^2 + \tfrac{8}{9}s + \tfrac{4}{27}$$

$$= \frac{4}{3} - \frac{\frac{8}{9}s+\frac{4}{27}}{(s+\frac{1}{3})^2} = \frac{4}{3} + \frac{A}{(s+1/3)^2} + \frac{B}{s+\frac{1}{3}}$$

$$A = -\frac{\frac{8}{9}(-\frac{1}{3}) + \frac{4}{27}}{1} = \frac{-4}{27}$$

$$B = \frac{d}{ds}\left(-\frac{8}{9}s + \frac{4}{27}\right)\Big|_{s=-\frac{1}{3}} = -\frac{8}{9}$$

$$\boxed{y(t) = \frac{4}{3} - \frac{4}{27}te^{-\frac{1}{3}t} - \frac{8}{9}e^{-\frac{1}{3}t}}$$

7. Thought question: If you are given a circuit, whose input is not specified (say it is an unknown voltage source), is it possible to find the circuit's system transfer function? Explain.

Yes, in fact we did this in 5a) above. A transfer function is a property of the system (which is usually a circuit in this class), and is completely independent of any particular input. Think of the system, as described by the transfer function, as a verb that acts upon the noun that is its input (a signal), producing another noun (signal).