P1 **Concept:** Equivalent system representations

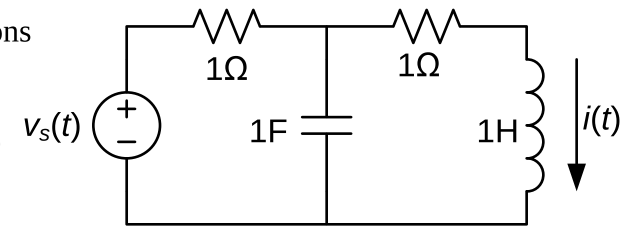
Find: Given the circuit with input $v_s(t)$, output $i(t)$ find:

a) system transfer function $H(s)$

b) s-plane plot

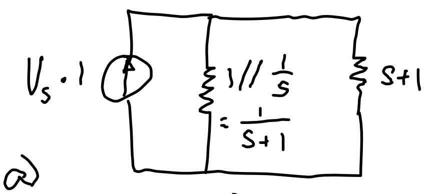
c) differential equation

d) impulse response $h(t)$



Hint: Its impulse response oscillates with a frequency of ω = 1 rad/sec

First, consider approach. Mesh (2 unknowns) beats nodal (2 unknowns) because result in terms of I. But even easier is to first use source transforms / Z simplification



by I divider

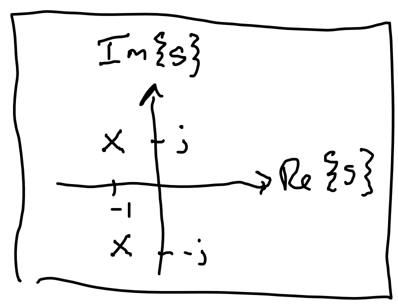
$$I = V_s\left(\frac{\frac{1}{s+1}}{\frac{1}{s+1} + s+1}\right) = V_s\left(\frac{1}{1+(s+1)^2}\right)$$

a)

$$H(s) = \frac{I(s)}{V_s(s)} = \frac{1}{(s+1)^2+1}$$

$$\boxed{H(s) = \frac{1}{s^2+2s+2}}$$

b) no zeros, poles: $s = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm j$



c) $\frac{I(s)}{V_s(s)} = \frac{1}{s^2+2s+2}$ cross mult

$$I(s)\left[s^2+2s+2\right] = V_s(s)$$

$$\boxed{i'' + 2i' + 2i = v_s}$$

d) could multiply out, find complex roots, etc., but simpler to see $\frac{1}{(s+1)^2+1}$ in table has the transform

$$\boxed{h(t) = e^{-t}\sin(t)\,u(t)}$$