1. If $x(t) = u(t)$ and $h(t) = 2e^{-t}u(t)$
Find y(t) using the Convolution integral equation.

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$

$$= \int_{-\infty}^{\infty} u(\tau)2e^{-(t-\tau)}u(t-\tau)\,d\tau$$

$u(\tau)$ turns on for $\tau > 0$
$u(t-\tau)$ turns on for $\tau < t$
$u(\tau)u(t-\tau)$ turns on for $\begin{cases} 0 \le \tau < t & \text{if } t \ge 0 \\ \text{always off} & \text{if } t < 0 \end{cases}$

$$= \int_0^t 2e^{-(t-\tau)}\,d\tau \; u(t)$$

$$= 2e^{-t}\int_0^t e^{\tau}\,d\tau \; u(t)$$

$$= 2e^{-t}\left[e^{t} - e^{0}\right]u(t)$$

$$= 2\left[e^{0} - e^{-t}\right]u(t)$$

$$= \boxed{2(1-e^{-t})u(t)}$$

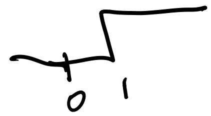
2. Above problem but find $y(t)$ using Laplace Transforms.

$$Y(s) = X(s)H(s)$$

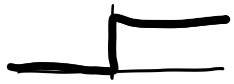
$$= \frac{1}{s}\cdot\frac{2}{s+1} = \frac{\cancel{A}\,2}{s} + \frac{\cancel{B}\,-2}{s+1}$$

$y(t) = \boxed{(2-2e^{-t})u(t)}$ checks with above

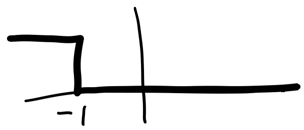
3. $x(t) = u(t), h(t) = u(t-1)$. Find $y(t)$ using graphical convolution.



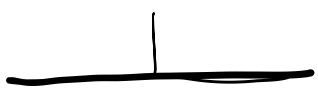
① Plot $x(\tau)$



② To find $y(0)$, plot $h(-\tau)$ e.g. "flip"



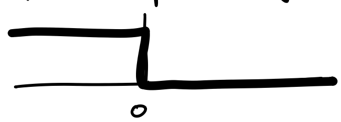
③ Mult $x(\tau) \cdot h(-\tau)$



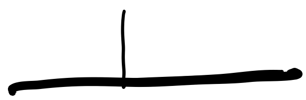
④ And integrate (find the area) $=0$

$y(t=0)=0$

② To find $y(1)$, plot $h(1-\tau)$ eg. "slide" the $h(-\tau)$ plot to right



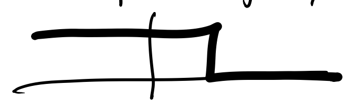
③ Mult $x(\tau) \cdot h(1-\tau)$



④ And integrate above $=0$

$y(t=1)=0$

② To find $y(2)$, plot $h(2-\tau)$, eg "slide" $h(-\tau)$ plot to right by 2



③ Mult $x(\tau) \cdot h(2-\tau)$

④ Integrate above $=1$

$y(t=2)=1$

As you continue the pattern above, you will see $y(t)=0$ for $t \leq 1$, then linearly grows with slope of 1 for $t \geq 1$, eg $y(t=3)=2$, $y(t=4)=3$ or more compactly, $\boxed{y(t) = (t-1)u(t-1)}$ or