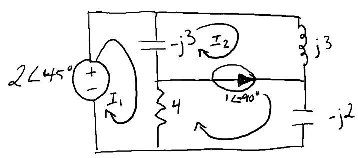
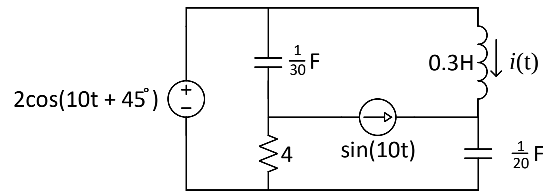
1. Find i(t) using mesh





Method 1: brute-force Mesh Current Analysis. Three meshes, so 3x3 set of equations.

KVL I_1: $-2\angle 45° + (I_1 - I_2)(-j3) + (I_1 - I_3)4 = 0 \Rightarrow \underline{I_1(4-j3) + I_2(j3) + I_3(-4) = 2\angle 45°}$

KVL supermesh (right mesh): $I_2(j3) + I_3(-j2) + (I_3 - I_1)4 + (I_2 - I_1)(-j3)$

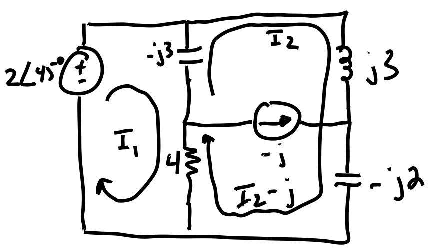
$\Rightarrow I_1(-4+j3) + I_2\cancel{(j3-j3)} + I_3(4-j2) = 0 \Rightarrow \underline{I_1(-4+j3) + I_3(4-j2) = 0}$

Extra eqn for Supermesh: $I_3 - I_2 = 1\angle -90° \Rightarrow \underline{I_2(-1) + I_3 = 1\angle -90°}$

$$\begin{bmatrix} 4-j3 & j3 & -4 \\ -4+j3 & 0 & 4-j2 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 2\angle 45° \\ 0 \\ 1\angle -90° \end{bmatrix} \Rightarrow \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 4.15\angle -61.9° \\ 3.70\angle -67.5° \\ 4.63\angle -72.2° \end{bmatrix} \Rightarrow \boxed{i_2(t) = 3.70\cos(10t - 67.5°)\,A}$$

Smart tip: recognize $1\angle -90° = -j$ which is easier to put into the calculator

Method 2: Smarter: recognize we could do it with just 2 meshes; I_1 and a supermesh around the whole right side named I_2 and I_2+the current source



KVL I_1: $-2\angle 45° + (I_1 - I_2)(-j3) + (I_1 - (I_2 - j))4 = 0$

$\Rightarrow I_1(4-j3) + I_2(-4+j3) = 2\angle 45° - j4$

KVL right side: $-j3(I_2 - I_1) + j3 I_2 - j2(I_2 - j) + 4(I_2 - j - I_1) = 0$

$\Rightarrow I_1(-4+j3) + I_2(-j3 + j3 - j2 + 4) + j^2 2 - j4 = 0$

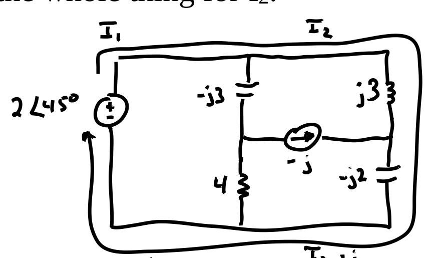
$= I_1(-4+j3) + I_2(4-j2) = 2 + j4$

$$\begin{bmatrix} 4-j3 & -4+j3 \\ -4+j3 & 4-j2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 2\angle 45° - j4 \\ 2 + j4 \end{bmatrix}$$

$\Rightarrow I_2 = 3.70\angle -67.5°$

$\Rightarrow \boxed{i_2(t) = 3.70\cos(10t - 67.5°)\,A}$

Method 3: Smartest: recognize that just one supermesh around the outside would solve the whole thing for I_2.



KVL outside: $-2\angle 45° + j3 I_2 - j2(I_2 - j) = 0$

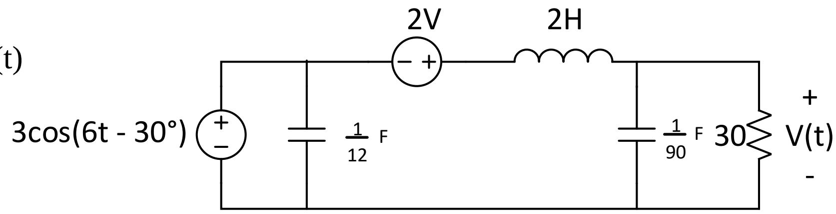
$\Rightarrow I_2(j3 - j2) - 2\angle 45° + j^2 2 = 0$

$\Rightarrow I_2 j = 2\angle 45° + 2$

$\Rightarrow I_2 = \dfrac{2 + 2\angle 45°}{j} = 3.70\angle -67.5°$

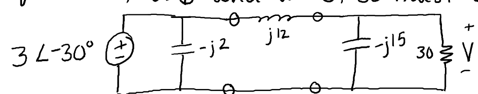
$\Rightarrow \boxed{i_2(t) = 3.70\cos(10t - 67.5°)\,A}$

2. Find *v*(t)

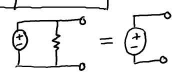


2 different frequencies, w=6 and w=0, so must use Superposition

① Left V

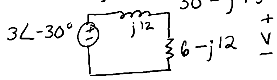


Simplifying to left of leftmost circles (recall



and to right of rightmost (V exists outside transformed part - not destroyed)

$30 // -j15 = \frac{(30)(-j15)}{30-j15} = 6-j12$



$V = 3\angle -30^\circ \frac{6-j12}{6-j12+j12}$ by V divider

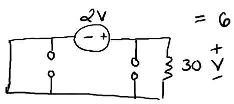
$= 3\angle -30^\circ \frac{6-j12}{6}$ by alg

$= 3\angle -30^\circ (1-j2)$ by alg

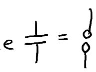
$= 6.71\angle -93.4^\circ$ by calc

$= 6.71 \cos(6t - 93.4^\circ)$ V

② Top V



Since $\frac{1}{j\omega C} = -j\infty$ for $w=0$ ⇒ open or since

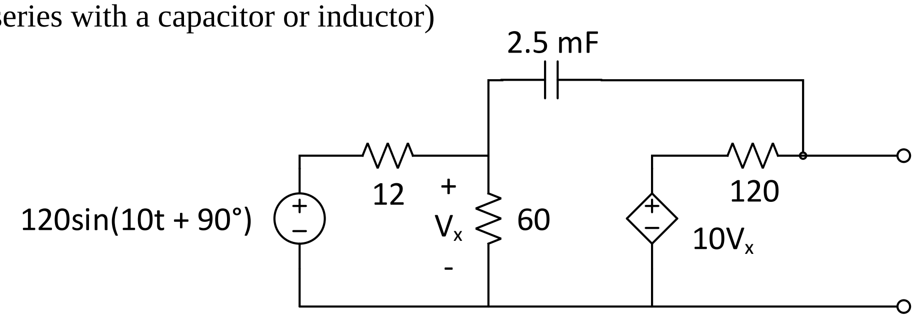


at DC

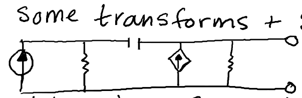
V=2V by inspection

$\boxed{V(t) = 6.71 \cos(6t - 93.4^\circ) + 2V}$

3. Find Thevenin equivalent in the time-domain (i.e. with an impedance made of a resistor in series with a capacitor or inductor)



Plan: ① Can we do some transforms + Simplification?

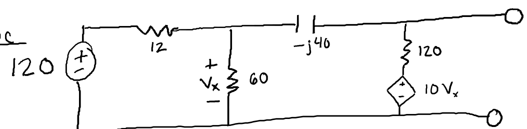


no, not all the way or we will loose V_x

② Can we find V_{oc} then zero all sources to find Z_{eq}?
no, because of dependent source

③ Find V_{oc}, then I_{sc}, then divide to find Z_{eq}
(If all sources were dependent, we would have to use a test source!)

Ⓐ V_{oc}



Mesh ⇒ 3 unknown meshes
Nodal ⇒ 2 unknown nodes ☆

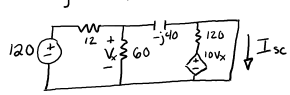
KCL @ V_x: $\frac{V_x - 120}{12} + \frac{V_x}{60} + \frac{V_x - V_{oc}}{-j40} = 0 \Rightarrow V_x\left[\frac{1}{12} + \frac{1}{60} + \frac{1}{-j40}\right] + V_{oc}\left(\frac{1}{j40}\right) = 10$

KCL @ V_{oc}: $\frac{V_{oc} - V_x}{-j40} + \frac{V_{oc} - 10V_x}{120} = 0 \Rightarrow V_x\left[\frac{1}{j40} - \frac{1}{12}\right] + V_{oc}\left[\frac{1}{-j40} + \frac{1}{120}\right] = 0$

$$\begin{bmatrix} \frac{1}{12} + \frac{1}{60} - \frac{1}{j40} & \frac{1}{j40} \\ \frac{1}{j40} - \frac{1}{12} & \frac{1}{-j40} + \frac{1}{120} \end{bmatrix} \begin{bmatrix} V_x \\ V_{oc} \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} V_x \\ V_{oc} \end{bmatrix} = \begin{bmatrix} 253\angle 34.7^\circ \\ 835\angle -20.2^\circ \end{bmatrix}$$

$$\boxed{V_{oc} = 835\angle -20.2^\circ \text{ V}}$$

Ⓑ I_{sc}



Mesh: 3 unknown meshes
Node: 1 unknown node ☆

KCL @ V_x: $\frac{V_x - 120}{12} + \frac{V_x}{60} + \frac{V_x}{-j40} = 0 \Rightarrow V_x\left(\frac{1}{12} + \frac{1}{60} + \frac{1}{-j40}\right) = \frac{120}{12} \Rightarrow V_x = \frac{10}{\frac{1}{12} + \frac{1}{60} + \frac{j}{40}}$ since $\frac{1}{-j} = j$ $= \underline{97\angle -14^\circ \text{V}}$

To find I_{sc} now that V_x is known, many ways. One is:

KCL upper-right: $I_{sc} + \frac{0 - 10V_x}{120} + \frac{0 - V_x}{-j40} = 0$

$I_{sc} = V_x\left(\frac{10}{120} + \frac{j}{40}\right)$ since $\frac{1}{-j} = j$

$= (97\angle -14^\circ)\left(\frac{1}{12} + \frac{j}{40}\right)$

$I_{sc} = 8.44\angle 2.66^\circ$ A

Ⓒ $Z_{eq} = \frac{V_{oc}}{I_{sc}} = \frac{835\angle -20.2^\circ}{8.44\angle 2.66^\circ} = 91.2 - j\,38.4$

$Z_C = -j\,38.4 = \frac{1}{j\omega C} = \frac{-j}{10C} \Rightarrow \cancel{-j} = (\cancel{-j}38.4)(10C) \Rightarrow C = \frac{1}{(38.4)(10)} = 2.6\text{ mF}$

835cos(10t -20.2°) V