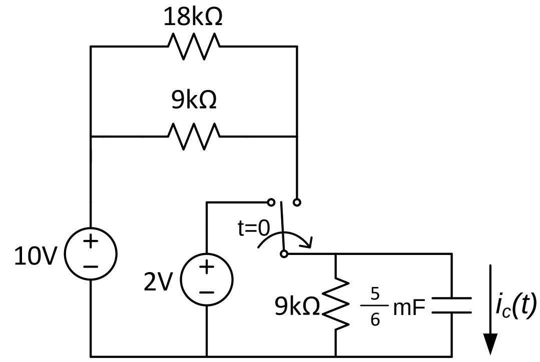
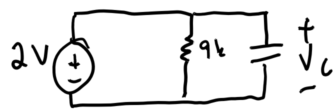
Problem 1: First-Order Circuits



a) Find $i_c(t)$ for *all* time. Hint: nice fractions.

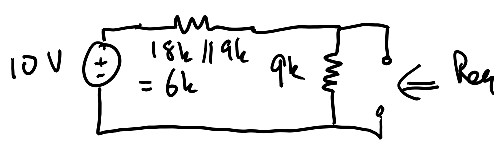
b) Find and sketch the current through the capacitor, $i_c(t)$ over five time constants τ. Label important values on both the v and t axes.

① $t<0$ Find v_{c0}

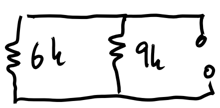


$\boxed{V_c = 2V \text{ by inspection}}$

② $t>0$ Find τ



recall impedance of $V_{source} = 0$



$\Leftarrow R_{eq} = 6k \| 9k = \frac{54}{15}k = \frac{18}{5}k$

$\tau = R_{eq}C = \left(\frac{18}{5}k\right)\left(\frac{5}{6}m\right) = \boxed{3}$ seconds

③ $t=\infty$ Find V_∞

$V_\infty = 10\frac{9k}{6k+9k} = 10\left(\frac{9}{15}\right) = \boxed{6V}$

④ $v_c(t) = V_\infty + (V_0 - V_\infty)e^{-t/\tau}$

$\boxed{= 6 - 4e^{-t/3}\ V}$

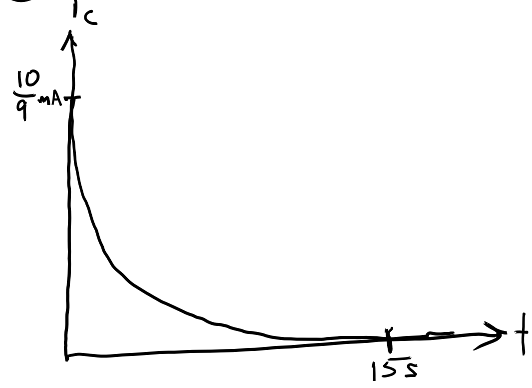
⑤ $i = Cv_c' = \left(\frac{5}{6}m\right)\left(\frac{4}{3}e^{-t/3}\right)$

$\boxed{i(t) = \frac{10}{9}e^{-t/3}\ mA}$

from $t<0$ diagram

⑥ $\boxed{i(t) = \begin{cases} 0A, & t<0 \\ \frac{10}{9}e^{-t/3}\ mA, & t\geq 0 \end{cases}}$

ⓑ Plot i_c

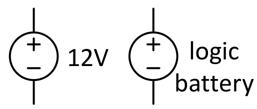


Problem 2: Laboratory-Based Design Problem

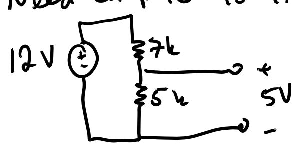
Your senior design robot is having problems. It works perfectly sometimes, but after about an hour of testing it starts shaking and acting as if it is having seizures. A few of you realize the problem is related to the logic battery pack powering the microcontroller; when it discharges below 5V it can no longer provide the power required by the microcontroller, which then behaves erratically. Help fix the problem by designing a circuit that lights a very bright warning LED when the logic battery pack drops below 5V. You may use the following components:

- 12V regulated voltage source devoted to your circuit (its voltage does not change)
- The logic battery pack output that is 6V when fully charged
- A bright red warning LED that drops 2.2V and requires 20mA
- A comparator
- Any number of resistors of any value

The start of the circuit is given:

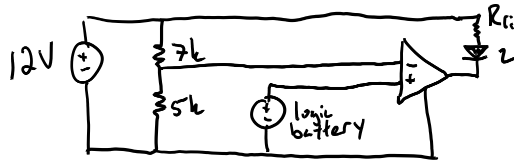


Need computer to trigger at 5V, so need to create 5V from 12V source. Use $V_{divider}$:



here I used 1kΩ to drop every volt, so that's 7k on top for the 7V it must drop and 5k on bottom to drop the remaining 5V

$R_{limit} = \frac{12-2}{0.02} = 500\Omega$



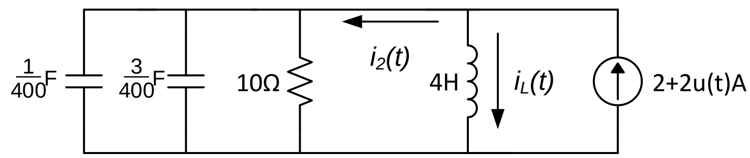
Check: for $V_{logic} > 5V$, $V^- < V^+$, output disconnects, light off

for $V_{logic} < 5V$, $V^- > V^+$, output grounds, light on

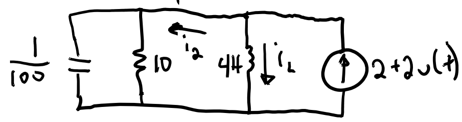
Problem 3: Second-Order Circuits

a) Given the circuit shown to the right, find $i_L(t)$, for $t>0$:

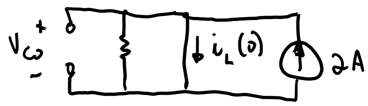
b) Find $i_2(t)$, for $t>0$:



First simplify C's in parallel

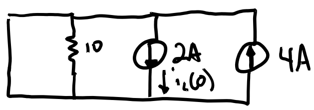


① $t<0$ Find $v_c(0)$, $i_L(0)$



by inspection $\boxed{\begin{array}{l} i_L(0) = 2A \\ v_c(0) = 0V \end{array}}$ since shorted

② $t=0^+$ Find $i_L(0^+)$, $i_L'(0^+)$

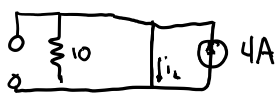


by inspection $\boxed{i_L(0^+) = 2A}$

$v_L(0^+) = L\, i_L'(0^+)$ but $V_L(0^+) = 0$ since shorted, so

$\boxed{i_L'(0^+) = 0\,A/s}$

③ $t=\infty$ Find $i_L(\infty)$



by inspection, $i_L(\infty) = 4A$

④ Find α, ω_0 and form of $i_L(t)$

$\alpha = \frac{1}{2RC}$ since parallel $= \frac{1}{2 \cdot 10 \cdot 1/100} = \frac{1}{2/10} = \boxed{5}$

$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4 \cdot \frac{1}{100}}} = \frac{1}{\sqrt{1/25}} = \frac{1}{1/5} = \boxed{5}$

$\alpha = \omega_0 \Rightarrow$ critically damped

$i_L(t) = C_1 e^{-\alpha t} + C_2 t\, e^{-\alpha t} + i_\infty$

$= C_1 e^{-5t} + C_2 t\, e^{-5t} + 4$

⑤ Match IC's

$i_L(0) = C_1 + 4 = 2 \Rightarrow C_1 = -2$

$i_L(t) = -2e^{-5t} + C_2 t e^{-5t} + 4$

$i_L'(t) = 10 e^{-5t} + C_2(-5) t e^{-5t} + C_2 e^{-5t} \Rightarrow i_L'(0) = 10 + C_2 = 0 \Rightarrow C_2 = -10$

$\boxed{i_L(t) = -2e^{-5t} - 10te^{-5t} + 4A}$

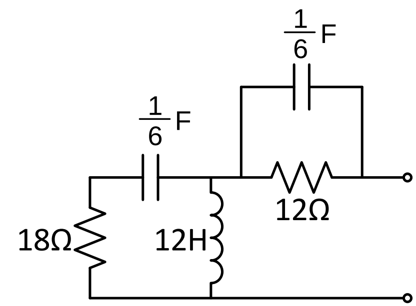
b) Find i_2, $t \geq 0$: could repeat all of above, but smarter:

by KCL, $i_2 = i_s - i_L = 4 - i_L$

$= \boxed{2e^{-5t} + 10te^{-5t} A}$

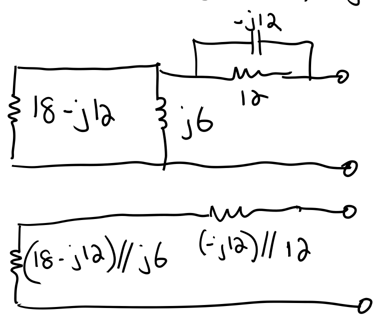
Problem 4: Phasors

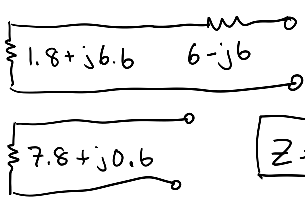
a) Given the circuit shown to the right, find the complex impedance, in rectangular form, between the circled terminals for a frequency of ω= ½ rad/s.



$Z_C = \frac{1}{j\omega C} = \frac{1}{j(\frac{1}{2})(\frac{1}{6})} = \frac{-j}{1/12} = -j12$

$Z_L = j\omega L = j(\frac{1}{2})(12) = j6$





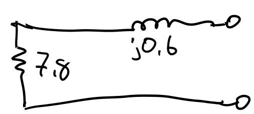
$\boxed{Z = 7.8 + j0.6\ \Omega}$

$(18 - j12)//j6 = \frac{(18 - j12)(j6)}{18 - j12 + j6} = 1.8 + j6.6$

$(-j12)//12 = \frac{(12)(-j12)}{12 - j12} = 6 - j6$

b) Find two components that, when connected in series, form the Thevenin equivalent to the entire circuit at that frequency in part a) of ω= ½ rad/s.

$Z = 7.8 + j0.6 =$

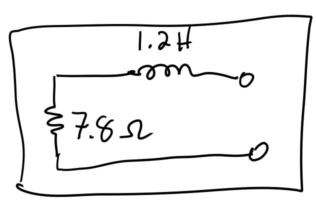


$Z_L = j0.6 = j\omega L$

$\Rightarrow j0.6 = j(\frac{1}{2})L$

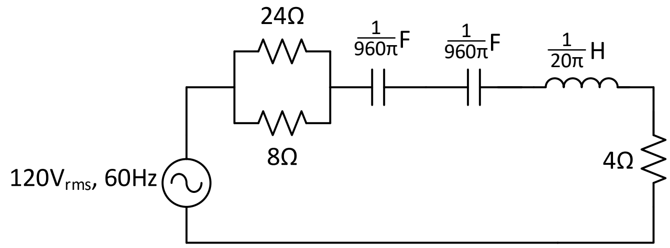
$\Rightarrow 0.6 = \frac{1}{2}L$

$\Rightarrow L = 1.2$



So this is the Thev equiv of the above complicated circuit for inputs of $\omega = \frac{1}{2}$ rad/s

Problem 5: Complex Power



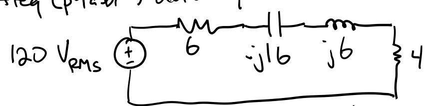
a) Find the real power P and complex power **S** power delivered to the 4Ω resistor.

$Z_R = 24 // 8 = 6\,\Omega$, $Z_{C1} = \frac{1}{j\omega C_1} = \frac{1}{j(2\pi 60)(\frac{1}{960\pi})} = \frac{1}{j(1/8)} = -j8$

$Z_{Ceq} = Z_{C1} + Z_{C2}$ since in series $= -j8 + -j8 = -j16$

$Z_L = j\omega L = j2\pi 60 \left(\frac{1}{20\pi}\right) = j6$

In freq (phasor) domain,



$\underline{V}_4 = 120 \cdot \frac{4}{6 - j16 + j6 + 4} = 120\left(\frac{4}{10 - j10}\right) = 24 + j24 \text{ V}_{RMS}$

$\underline{I}_4 = \frac{\underline{V}_4}{4} = 6 + j6 \text{ A}_{RMS}$

$\underline{S}_4 = \underline{V}_4 \cdot \underline{I}_4^* = (24+j24)(6+j6)^* = (24+j24)(6-j6) = \boxed{288 \text{ VA}}$

$P = \text{Real}\{\underline{S}_4\} = \boxed{288\text{W}}$ since R's impedance is purely real, all its complex power is real

b) Determine the power factor seen by the source and correct it to 1 by using a capacitor or inductor in series with the source.

Power factor : $\underline{S}_{source} = \underline{V}_{SOURCE} \cdot \underline{I}^*_{SOURCE}$ ← found in part a)

$= (120\angle 0^\circ)(6+j6)^* = 120(6-j6) = 720 - j720 = \underset{\uparrow S}{1018} \underset{\uparrow pf = \cos(-45^\circ)}{\angle -45^\circ}$

$pf = \cos(-45^\circ) = \boxed{0.707} = \frac{1}{\sqrt{2}}$

Correction : If the problem said to use C in parallel with source, could use plug-n-chug equation in text. But in series? Will have to think ...
A pf of 1 means the load looks purely resistive, with no imaginary component.
Our load looks like $6 - j16 + j6 + 4 = 10 - j10\ \Omega$.
So, a series impedance of $+j10$ would make the load look purely resistive and solve the problem. Positive, so will be an inductor.

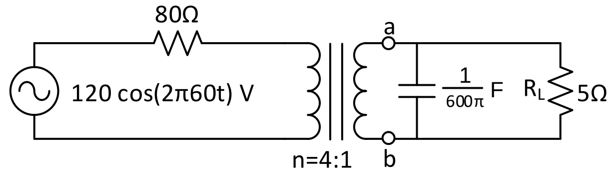
$Z_L = +j10 = j\omega L$

$\Rightarrow j10 = j(2\pi 60) L$

$\Rightarrow L = \frac{j10}{j2\pi 60} = \frac{1}{2\pi 6} = \frac{1}{12\pi}$ H

Solution: put a $\boxed{L = \frac{1}{12\pi} H}$ in series with source for $pf = 1$

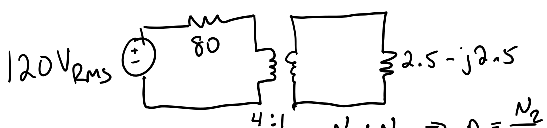
Problem 6: Transformers



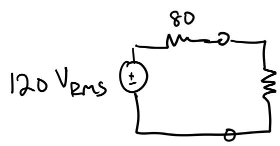
a) Circuit II cadets notice some dislodged ceiling tiles in the lab and, upon investigating, discover the circuit to the right hidden above the ceiling, right next to the First Class beer stash. The power source at the left indicates where it is plugged into a power outlet. The circuit to the right of a-b represents a solid-state Peltier cooling junction used to ice the beer. Suspecting foul play given the irrational choice of capacitor, they calculate the average power being absorbed by the load to the right of the terminals a-b (i.e. the load is both the capacitor and the resistor). Recall powerline frequency in the U.S. is 60Hz.

$Z_C = \frac{1}{j\omega C} = \frac{1}{j\cdot 2\pi 60 \cdot \frac{1}{600\pi}} = \frac{1}{j\,{}^1\!/_5} = -j5$

$Z_{LOAD} = Z_C \parallel Z_R = -j5 \parallel 5 = \frac{-j25}{5-j5} = 2.5 - j2.5$ (calculator used here)



$= N_1 : N_2 \Rightarrow n = \frac{N_2}{N_1} = \frac{1}{4}$



$\frac{Z_L}{n^2} = \frac{2.5-j2.5}{(1/4)^2} = 40 - j40$

To find the real power, first find the complex power phasor $\underline{S} = \underline{V}_{RMS} \cdot \underline{I}_{RMS}^*$

$\underline{V}_{RMS} = 120 \cdot \left(\frac{40-j40}{80+(40-j40)}\right) = 48 - j24\ V_{RMS}$, $\underline{I}_{RMS} = \frac{120}{80+(40-j40)} = 0.9 + j0.3\ A_{RMS}$

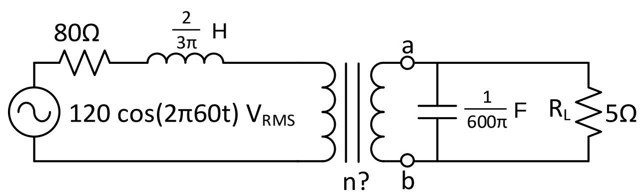
$\underline{S} = \underline{V}_{RMS} \cdot \underline{I}_{RMS}^* = (48-j24)(0.9-j0.3) = 36 - j36$ VA , $\boxed{P = \text{Real}\{\underline{S}\} = 36W}$

b) How much of the average power you calculated in part a) is being absorbed by the resistor, and how much is being absorbed by the capacitor?

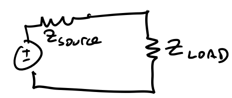
You could do the calculation in part a) for the capacitor + the resistor, but there is a smarter way. Capacitors cannot absorb average power (they absorb energy for 1/2 period + discharge it for the next 1/2, so average power absorbed = 0). $\Rightarrow \boxed{P_{cap} = 0W}$

That leaves the resistor to absorb the remaining 36 W calculated in part a). $\Rightarrow \boxed{P_{resistor} = 36W}$

c) The next day they discover the Seniors are trying to make the Peltier junctions work more efficiently by adding an inductor as shown below. Alas, they have failed to design the optimal transformer because they forgot their Circuits II skills. Help them find the correct transformer ratio *n* to optimize power delivery to the load. (Hint: it won't be a whole number. That's OK; you can design a, say, n=0.123 ratio transformer by noting 0.123 = 123/1000, so the primary has 123 windings for every 1000 in the secondary).



To optimize P_{LOAD}, $Z_{LOAD} = Z^*_{SOURCE}$ in this circuit:

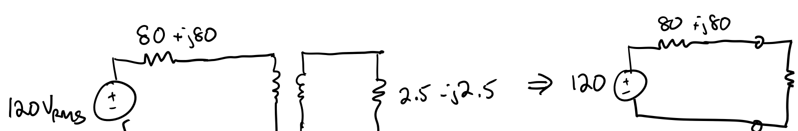


$Z_{LOAD} = \frac{Z_L}{n^2}$

$Z_{inductor} = j\omega L = j\cdot 2\pi 60 \cdot \frac{2}{3\pi} = j80$

$Z^*_{SOURCE} = \frac{Z_L}{n^2} \Rightarrow (80+j80)^* = \frac{2.5-j2.5}{n^2} \Rightarrow n^2 = \frac{2.5-j2.5}{80-j80} \Rightarrow \boxed{n = 0.177}$

d) To show off, the Circuits II students leave a note listing both the reactive and complex power dissipated by the modified circuit's load (i.e. the capacitor and the 5Ω resistor). What did the note say? Include units.



$\frac{2.5-j2.5}{n^2} = 80 - j80$

Find the complex power phasor first, $\underline{S} = \underline{V}_{rms}\cdot \underline{I}^*_{rms}$

$\underline{V}_{rms} = 120\,\frac{80-j80}{(80+j80)+(80-j80)} = 120\,\frac{80-j80}{160} = 60 - j60\ V_{rms}$

$I_{rms} = \frac{120}{(80+j80)+(80-j80)} = \frac{120}{160} = \frac{3}{4}\ A_{rms}$

$\underline{S} = \underline{V}_{rms}\cdot I^*_{rms} = (60-j60)\left(\frac{3}{4}\right) = \boxed{45 - j45\ \text{VA}}$ complex power of load

$Q = Im\{\underline{S}\} = \boxed{-45\ \text{VAR}}$ reactive power of load