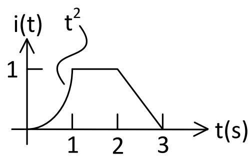
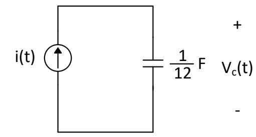
1)





Since $i(t) = \frac{1}{C} = 0$ for all $t<0$. C must be initially discharged so $V_c(t=0) = 0$

time Region	t_0	$V(t_0)$	$i(t)$	$V_c(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + V_c(t_0)$
$t<0$	—	0 V	0A	0V
$0 \le t \le 1$	0	0	t^2	$\frac{1}{1/12}\int_0^t \tau^2 d\tau + 0 = 12(\frac{1}{3}t^3) = 4t^3$
$1 \le t \le 2$	1	$4(1)^3 = 4$	1	$12\int_1^t d\tau + 4 = 12\tau\Big]_{\tau=1}^{t} + 4 = 12t - 12 + 4 = 12t - 8$
$2 \le t \le 3$	2	$12(2) - 8 = 16$	$-(t-3) = 3-t$	$12\int_2^t (3-\tau)d\tau + 16$ $= 12[3\tau - \frac{1}{2}\tau^2]_{\tau=2}^{t} + 16 = 12[(3t - \frac{1}{2}t^2) - (6-2)] + 16$ $= 12[3t - \frac{1}{2}t^2 - 4] + 16$ $= -6t^2 + 36t - 32$
$t \ge 3$	3	$-6(3^2) + 36(3) - 32 = 22$	0	$= 12\int 0\, d\tau + 22 = 22$

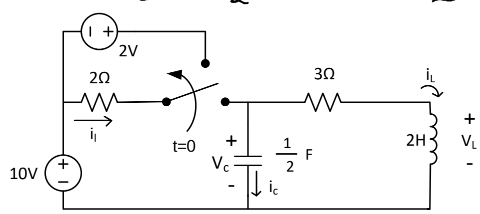
a) Find $v_c(t)$

$$\boxed{V_c(t) = \begin{cases} 0\text{ V} & , t<0 \\ 4t^3\text{ V} & , 0 \le t < 1 \\ 12t - 8\text{ V} & , 1 \le t < 2 \\ -6t^2 + 36t - 32\text{ V} & , 2 \le t < 3 \\ 22\text{ V} & , t \ge 3 \end{cases}}$$

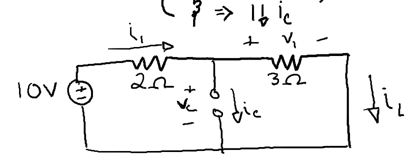
b) Find $W_c(2)$

$$W_c(2) = \frac{1}{2} C V_c^2(2) = \frac{1}{2}\left(\frac{1}{12}\right)(16)^2 = \frac{16^2}{24} = \boxed{10.7\text{J}}$$

2)

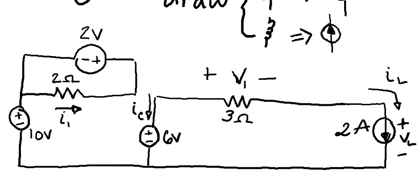


$t<0$ draw $\begin{cases} \text{capacitor} \Rightarrow \text{open}, +V_c - \\ \text{inductor} \Rightarrow \text{short}, i_c \end{cases}$



a) Find i_I, i_c, i_L, v_c, v_L, at both t = -3 and t = 0^+

$t = 0^+$ draw $\begin{cases} \text{capacitor} \Rightarrow \text{voltage source} \\ \text{inductor} \Rightarrow \text{current source} \end{cases}$



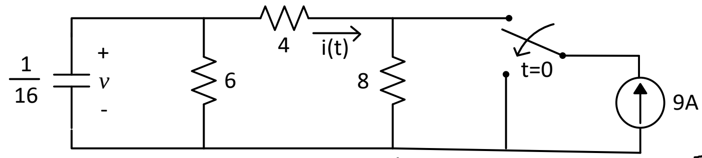
	$t<0$ (e.g. $t=-3$)	$t = 0^+$
i_1	2A ⇐ Ω's law	-1A ⇐ Ω's law
i_c	0A ⇐ open	-2A ⇐ current source
i_L	$2A \Leftarrow \frac{10V}{5\Omega}$	2A ⇐ current continuity $i_L(0^+) = i_L(0^-)$
V_c	$6V \Leftarrow \frac{3}{2+3} 10$	6V ⇐ voltage continuity $V_c(0^+) = V_c(0^-)$
V_L	0V ⇐ short	$0V \Leftarrow KVL + 3(2) + V_c - 6 = 0$

b) Find Q_c (t = -2), W_L (t = 0^+)

$Q(t=-2) = CV_c(t=-2) = (\frac{1}{2}F)(6V) = \boxed{3C}$

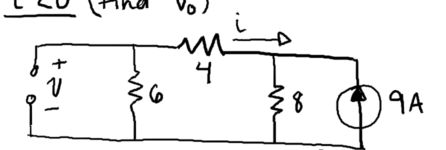
$W_L(t=0^+) = \frac{1}{2}L i_L^2(0^+) = \frac{1}{2}(2)(2)^2 = \boxed{4J}$

3) Find *v*(t) and *i*(t) for all time



to find $v(t)$ must first find V_0, V_∞, τ_1 To find $i(t)$, note $i(t) = \frac{v(t)}{4+8}$

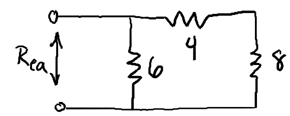
t<0 (find V_0)



by i divider, $i = -\frac{8}{8+10}\cdot 9 = -4A$

by Ω's law, $V_0 = -6i = 24V$

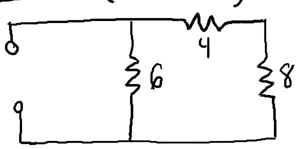
t>0 (find τ)



$R_{eq} = 6 // 12 = 4\Omega$

$\tau = R_{eq}C = (\frac{1}{16})4 = \frac{1}{4}$ sec

t=∞ (find V_∞)

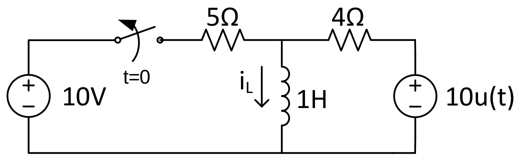


$V_\infty = 0$ by inspection (no source)

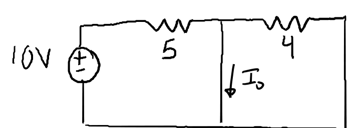
$$v(t) = \begin{cases} 24V, & t<0 \\ V_\infty + (V_0 - V_\infty)e^{-t/\tau}, & t>0 \end{cases} = \boxed{\begin{cases} 24V, & t<0 \\ 24e^{-4t}, & t>0 \end{cases}}$$

$$i(t) = \begin{cases} -4A, & t<0 \\ \frac{V_c}{4+8}, & t>0 \end{cases} = \boxed{\begin{cases} -4A, & t<0 \\ 2e^{-4t}A, & t\geq 0 \end{cases}}$$

4) Find v_L (t), t ≥ 0

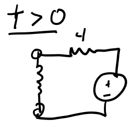


t<0

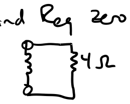


$I_0 = \frac{10V}{5\Omega} = \boxed{2A}$

t>0



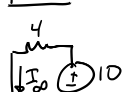
to find Req zero sources ⇒



Inductor sees 4Ω

$\tau = L/R = \boxed{1/4}$ s

t=∞



$I_\infty = \frac{10}{4} = \boxed{2.5A}$

$i_L(t) = I_\infty + (I_0 - I_\infty)e^{-t/\tau}$

$= 2.5 + (2-2.5)e^{-4t}$

$= \boxed{2.5 - 0.5e^{-4t}}$

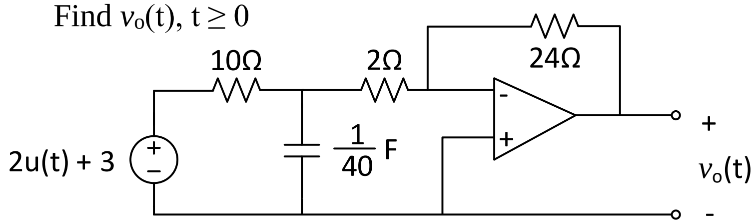
$v_L = Li'$

$= (1H)\frac{d}{dt}(2.5 - 0.5e^{-4t})$

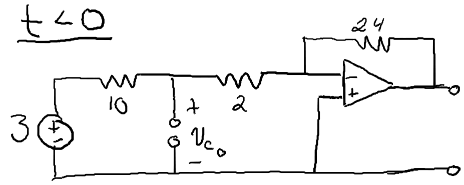
$= 0 - 0.5(-4)e^{-4t}$

$= \boxed{2e^{-4t}\ V}$

5) Find $v_o(t)$, $t \geq 0$



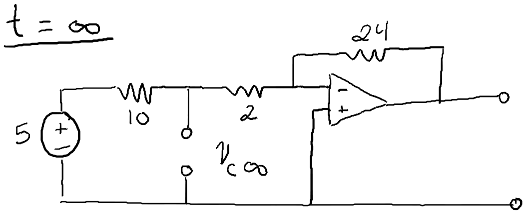
$\underline{t < 0}$



$$KCL @ V_c: \frac{V_c - 3}{10} + \frac{V_c - 0}{2} = 0$$

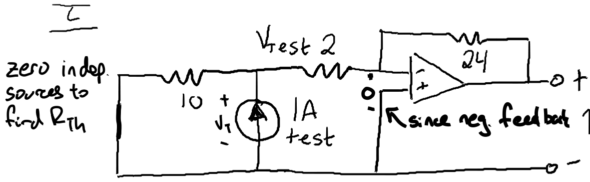
$$V_c(0^-) = \frac{1}{2}$$

$\underline{t = \infty}$



$$KCL @ V_c: \frac{V_c - 5}{10} + \frac{V_c - 0}{2} \Rightarrow V_\infty = \frac{5}{6}$$

$\underline{\tau}$

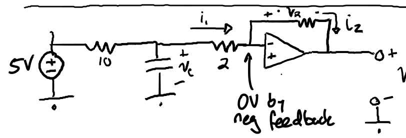


$$KCL \text{ above } 1A: \frac{V_{test}}{10} - 1 + \frac{V_{test} - 0}{2} = 0$$

$$\text{Solve for } V_{test} = \frac{5}{3} \Rightarrow R_e = \frac{V_{test}}{I_{test}} = \frac{5}{3}$$

$$\tau = R_e \cdot C = \frac{5}{3} \cdot \frac{1}{40} = \frac{1}{24}$$

$$V_c(t) = V_\infty + (V_0 - V_\infty) e^{-t/\tau} = \boxed{\frac{5}{6} - \frac{1}{3} e^{-24t}}$$ $\underline{\text{BUT}}$ we want V_o, not V_c



To find V_o given V_c, ① $i_1 = \frac{V_c - 0}{2}$

② $i_2 = i_1 = \frac{V_c}{2}$

③ $V_R = 24 \cdot i_2 = 24\left(\frac{V_c}{2}\right) = 12V_c$

④ $V_o = 0 - V_R = -12V_c = -12\left(\frac{5}{6} - \frac{1}{3}e^{-24t}\right)$

$= \boxed{-10 + 4e^{-24t}\ V}$