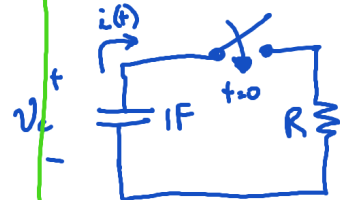


Singularity Functions

- Intuition
- Math: step, impulse, ramp
 $u(t)$ $\delta(t)$ $r(t)$
- Circuits!

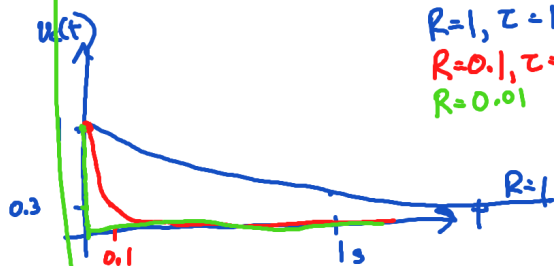
Intuition



$$V_c(0) = \underline{1V}$$

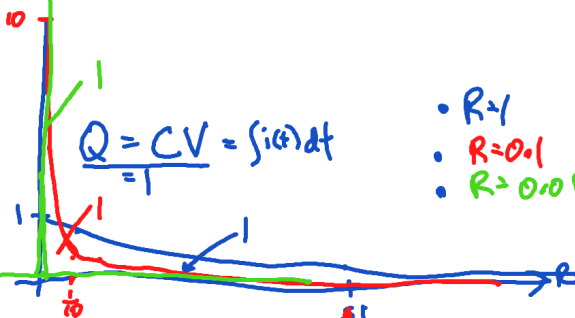
$$\lim_{R \rightarrow 0} \tau = R$$

$$V_c(t) = e^{-t/R} \quad \leftarrow v(0) = e^{-0/R} = e^0 = 1$$



$R=1, \tau=1s$
 $R=0.1, \tau=0.1$
 $R=0.01$

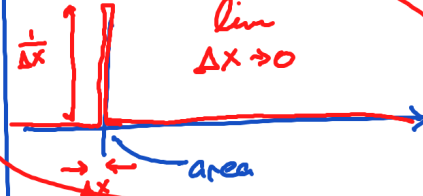
$$i(t) = \underline{CV'_c} = \frac{V}{R} = \frac{1}{R} e^{-t/R}$$



$$Q = CV = \int i(t) dt$$

- $R=1$
- $R=0.1$
- $R=0.01$

as $R \rightarrow 0$



$$\text{area} = \text{width} \cdot \text{height} = (\Delta x) \left(\frac{1}{\Delta x} \right) = 1$$

$$\delta(t) = 1$$

$$\delta(t) = \begin{cases} \infty, & t=0 \\ 0, & \text{otherwise} \end{cases}$$

Unit sample impulse $\delta(t)$



Math Review

Given plot $f(t)$



Move up by 2



Move right by 2



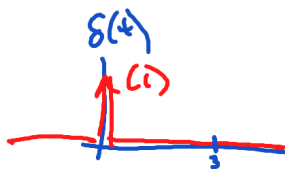
Flip horiz



Flip vertically



$\delta(t)$ Impulse



$2\delta(t-3)$



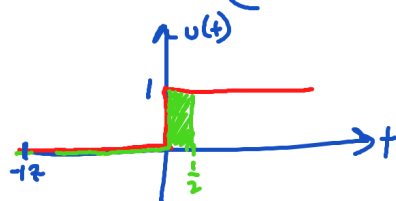
$$\delta(3) = 0$$

$$\delta(-1) = 0$$

$$\delta(0) = ? \infty$$

$u(t)$ step function

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

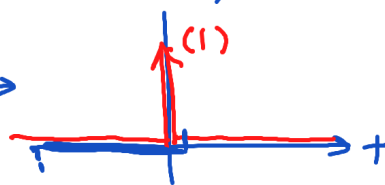


$$u(-2) = 0$$

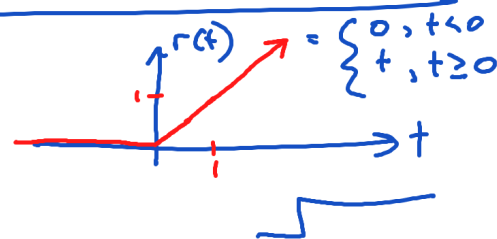
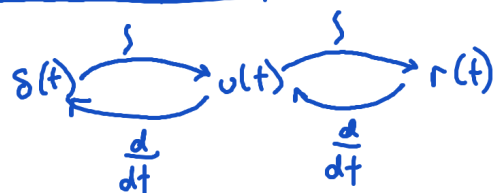
$$\frac{d}{dt}$$

$$\int dt$$

$\delta(t)$



$r(t)$ ramp function

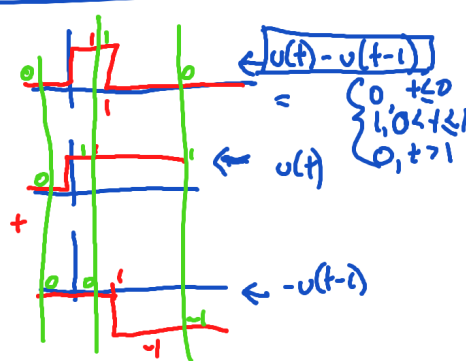


$$f(t) = \begin{cases} 0 & t < 1 \\ 2 & t \geq 1 \end{cases} = 2u(t-1)$$

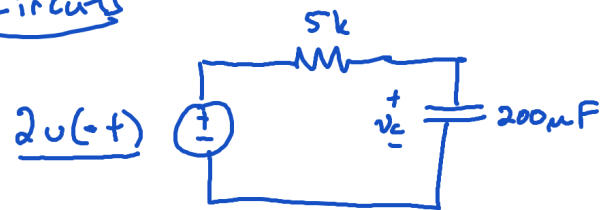


$$3u(t+4) + 2\delta(t-2)$$

$$u(t+4)$$



Circuits



Find $v_c(t)$

① V_{co} , $t < 0$

$t = -10$
 $2u(-10) = 2u(10) = 2$



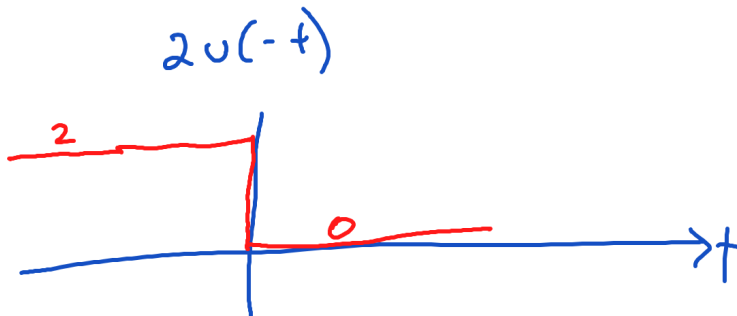
② $t > 0$

$\tau = RC = 5k \cdot 200\mu$
 $= 1000m = 1s$

③ V_{co} $t = \infty$



④ $v_c(t) = 2e^{-t} V$



$v(17) = 2u(-17) = 2 \cdot 0 = 0$
 $v(3) = 2u(-3) = 0$
 $v(-1) = 2u(1) = 2$