Obj

- Review
- Apparent Power
- Power Factor
- Leading / Lagging Power

Read

11.5 (11.6)

HW

PS 34 (not collected)

Leading / lagging

$\theta_I > \theta_v$ leading pf = capacitive

$\theta_I < \theta_v$ lagging pf = inductive

$\theta_I = \theta_v$ pf 1 = resistive

Last Time

Notation example: let $v(t) = 10\cos(3t + 40°)$

then $V_m = 10$ $\theta_v = 40$ $\mathbf{V} = 10\angle 40°$ $\mathbf{V}^* = 10\angle -40°$

$V_{RMS} = \frac{10}{\sqrt{2}} V_{RMS}$ $\mathbf{V}_{RMS} = \frac{10}{\sqrt{2}}\angle 40° V_{RMS}$

Average Power $= \frac{1}{T}\int_0^T p(t)\,dt$ any waveform

$= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_I) = \frac{1}{2} Re\{\mathbf{V}\cdot\mathbf{I}^*\}$

$= V_{RMS} I_{RMS} \cos(\theta_v - \theta_I) = Re\{\mathbf{V}_{RMS}\cdot\mathbf{I}_{RMS}^*\}$

sinusoids only

any component RLC

$\rightarrow V_{RMS} = \frac{V_m}{\sqrt{2}}$

$= S \cdot pf$

apparent power power factor

Apparent Power

- measured in VA not W {instantaneous power / average power ← apparent power
- $S = V_{RMS} \cdot I_{RMS}$
- scalar, not a phasor
- $P_{ave} = 0$ for C, L
  $S \neq 0$ for C, L
- what the power company charges you!

Power Factor pf

- $\cos(\theta_v - \theta_I) = \cos(\theta_I - \theta_v)$
- 1 for purely resistive load, 0 for purely capacitive or inductive
- $P_{ave} = S \cdot pf$
  ↑ what you are charged for
  ↑ what you can use

Example

across load $v(t) = 120\cos(2\pi 60 t - 20^\circ)\ V_{RMS}$

$i(t) = 4\cos(2\pi 60 t + 10^\circ)\ A_{RMS}$

analyze

- voltage measured in Volts?

$v(t) = 120\sqrt{2}\cos(2\pi 60 t - 20^\circ)\ V$

$i(t) = 4\sqrt{2}\cos(2\pi 60 t + 10^\circ)\ A$

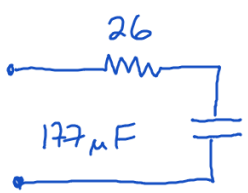
- apparent power $= S = V_{RMS} \cdot I_{RMS} = \underbrace{120}_{V_{RMS}} \cdot \underbrace{4}_{A_{RMS}} = 480\ VA$
- $pf = \cos(\theta_I - \theta_V) = \cos(10 - -20) = \cos(30^\circ) = 0.866$, leading
- leading or lagging? $\theta_I > \theta_V$

$10^\circ > -20^\circ$

leading

- $P_{ave} = S \cdot pf = (480)(0.866) = 417W$
- Load impedance $Z = \frac{V_{RMS}}{I_{RMS}} = \frac{120\angle -20^\circ}{4\angle 10^\circ} = 30\angle -30^\circ = \underline{26 - j15\ \Omega}$

Load

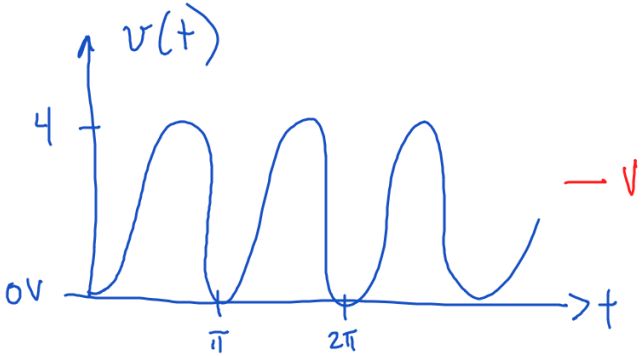


$\frac{-j}{\omega C} = -j15$

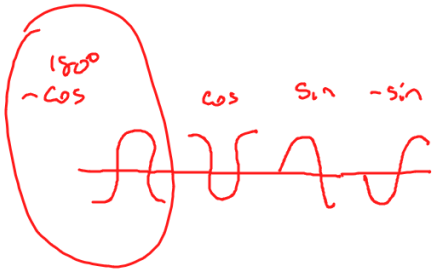
$\omega = 2\pi 60$

$C = 177\mu F$

HW Questions



— V_{rms} guess 2V



Find V_{Rms}

$v(t) = 2 + 2\cos(2t + \pm 180°)$

$\omega = 2\pi f = 2\pi \frac{1}{\pi} = 2$

$f = \frac{1}{\pi}$

$T = \pi$

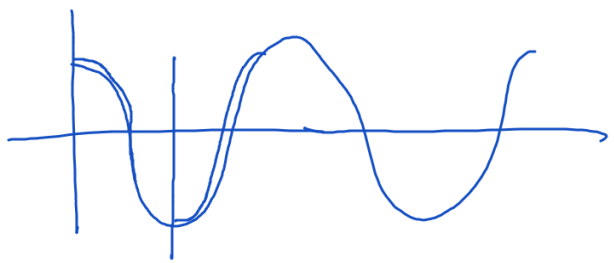
$\tan(x+180°) = -\tan(x)$

$\cos(x+180°) = -\cos(x)$

$\sin(x+180°) = -\sin(x)$

$v(t) = 2 + 2\cos(2t+180°)$

$= 2 - 2\cos(2t)$



$V_{Rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$

$= \sqrt{\frac{1}{\pi}\int_0^T (2-2\cos(2t))^2\,dt}$

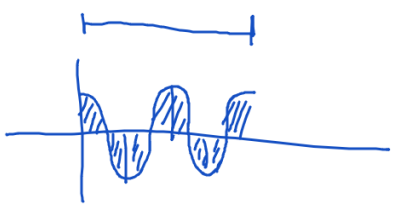
$V_{Rms}^2 = \frac{1}{\pi}\int_0^T 4 - 8\cos(2t) + 4\cos^2(2t)\,dt$

$= \frac{1}{\pi}\int_0^{T=\pi} 6 - 8\cos(2t) + 2\cos(4t)\,dt$

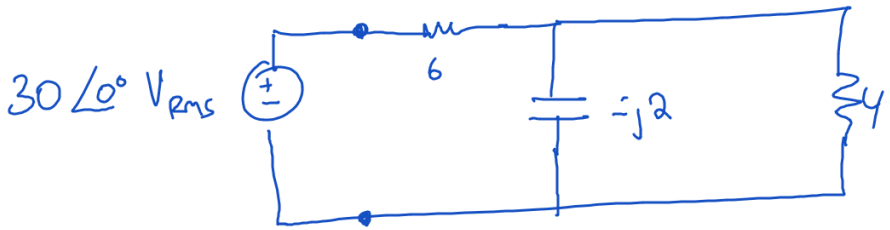
$= \frac{1}{\pi} 6\pi$

$= 6$

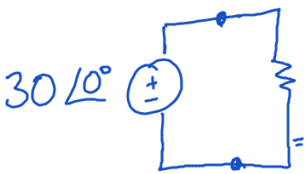
$V_{Rms} = \sqrt{6} = \boxed{2.45\ V_{Rms}}$



Collaborative Problem



a) Find pf of load as seen by the source. Include whether leading/lagging.



$= 6 + \frac{-j8}{4-j2}$

$= \underline{6.8 - j1.6}$

$V_{RMS} = 30\angle 0°$

$I_{RMS} = \frac{V_{RMS}}{Z} \quad \frac{30\angle 0°}{6.8 - j1.6} = \boxed{4.29\angle 13.2° \; A_{RMS}}$

$\theta_I > \theta_V$

$13.2° > 0°$

$pf = \cos(\theta_I - \theta_V)$

$= \cos(13.2° - 0°)$

$= \boxed{0.973 \text{ leading}}$ ✓

b) Find S delivered by source. Include units.

$S = I_{RMS} \cdot V_{RMS}$

$= (4.29)(30)$

$= \boxed{129 \text{ VA}}$

c) Find average power P_{ave} delivered by source. Include units.

$P_{ave} = S \cdot pf$

$= 129 \cdot 0.973$

$= \boxed{125 \text{ W}}$

pf: # leading/lagging

S: # VA

P_{ave}: # W