A. Instantaneous Power

1. $p(t) = i(t)\,v(t)$
2. if sinusoid then $p(t)$ is
   - 2x frequency of $i(t), v(t)$
   - DC part

   $\frac{1}{2}(DC + \cos(2f\,2\pi t + \theta))$
3. In general $p(t) \neq 0$ for R, C, L

   always a function of time

B. Average Power (of a periodic waveform of T)

1. $\boxed{\frac{1}{T}\int_0^T p(t)\,dt}$ it is a number not a function of time

   ↳ any waveform
2. For sinusoids, $\boxed{\begin{aligned} P_{ave} &= \frac{1}{2} \mathbf{V}_M I_M \cos(\theta_V - \theta_I) \\ &= \frac{1}{2} \mathrm{Re}[\mathbf{V} \cdot \mathbf{I}^*] \end{aligned}}$ only sinusoids

   $v(t) = V_M \cos(\omega t + \theta_v) = \mathbf{V}$
3. $\boxed{\begin{aligned} &P_{ave} \text{ for } L, C = 0W \\ &\text{for } R \geq 0W \text{ (not negative)} \end{aligned}}$ any waveform

C. Root Mean Square Voltage V_{RMS} (I_{RMS})

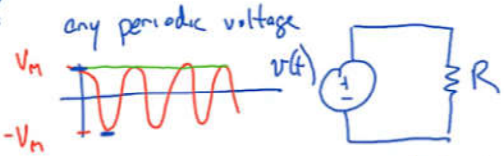
number like P_{ave}
not function like $p(t)$

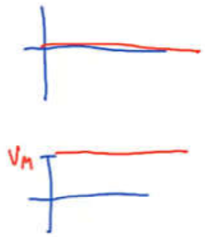
1. $V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$ (or $i(t)$) any waveform
2. $\frac{V_m}{\sqrt{2}}$ for sinusoids
3. $P_{ave} = V_{RMS} I_{RMS} \cos(\Delta\theta)$ sinusoid

   $= V_{RMS} I_{RMS}$ across any resistor any waveform

RMS Voltage, RMS Current

What? any periodic voltage





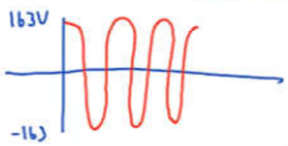
What is the DC voltage that could replace $v(t)$ so that P_{ave} of R stays the same?

Answer

$$\boxed{V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}}$$

also I_{RMS} any waveform

Note

- For sinusoids $\boxed{V_{RMS} = \frac{V_m}{\sqrt{2}}}$
- 

$\Rightarrow V_{RMS} = \frac{163\ \text{Volts}}{\sqrt{2}} = 120\ V_{RMS}$
- Units V_{RMS}, not V
- Sinusoid, V_{RMS} (scalar) $\mathbf{V}_{RMS}$ (phasor), ex
  - $V_{RMS} = 120\ V_{RMS}$ ← scalar. We don't know phase,
  - $\mathbf{V}_{RMS} = 120\angle 45°\ V_{RMS}$ ← phasor (complex) We do know phase

So what? Takes away $\frac{1}{2}$ factor for P_{ave} sinusoids

$$\boxed{P_{ave} = \frac{1}{2} V_m I_m \cos(\theta_V - \theta_I) = V_{RMS} \cdot I_{RMS} \cos(\Delta\theta)}$$

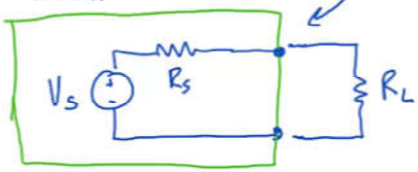
$$\boxed{= \frac{1}{2} \text{Re}[\mathbf{V} \cdot \mathbf{I}^*] = \text{Re}[\mathbf{V}_{RMS} \cdot \mathbf{I}^*_{RMS}]}$$

Lots good examples in text

Max. Average Power Transfer

Recall Circuits I

Thev equiv.



What R_L be to dissipate max power?
$R_L = R_s$!

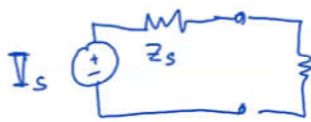
$$P_{max\ LOAD} = \frac{V_s^2}{4R}$$

$R_L = \infty\,\Omega \Rightarrow I = 0 \Rightarrow P = 0$

$R_L = 0\,\Omega \Rightarrow V = 0 \Rightarrow P = 0$

$P = I \cdot V$

Circuits II

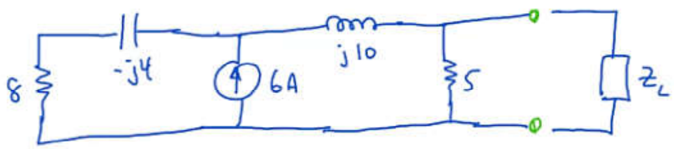


$$\boxed{Z_L = Z_s^*} \Rightarrow \boxed{P_{max\ \underline{average}\ load} = \frac{|\mathbf{V}_s|^2}{8\,\mathrm{Re}\{Z_s\}}}$$

Why 8, not 4? Because $P = \frac{1}{2}\mathrm{Re}[\mathbf{V} \cdot \mathbf{I}^*]$

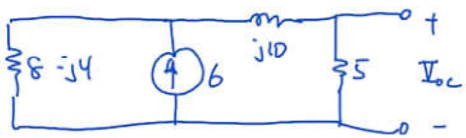
Ex Max Power Transfer

Find Z_L to receives the max power
What is this max power?

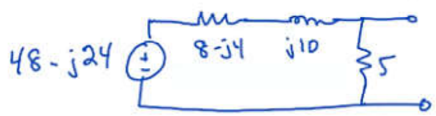


① Find Thev to left

ⓐ V_{oc}



Nodal: 2 unknowns
Mesh: 1 supermesh + addl step

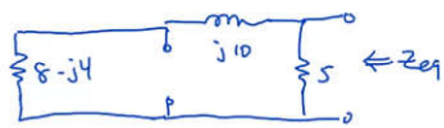


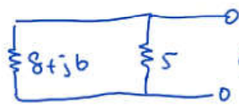
$$V_{oc} = (48-j24)\frac{5}{13+j6}$$

$$= 18.7\angle -51.3°$$

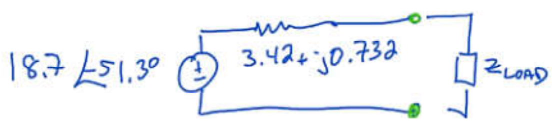
Isc

ⓑ Z_{eq}





$$\Leftarrow Z_{eq} = (8+j6)\,\|\,5 = \frac{(8+j6)5}{13+j6} = 3.42+j0.732$$



Video cut this off; should be
$-j/wc = -j0.732$
$1/wc = 0.732$
$C = 1/(0.732\ w)$

② $Z_L = Z_s^*$

$$= \boxed{3.42 - j0.732} =$$

$$Z_C = \frac{1}{j\omega C} = \frac{-j}{\omega C} = -j0.7$$

$$\frac{1}{\omega C} = 0.7$$

$$P_{max\ ave\ load} = \frac{|V_s|^2}{8\,\mathrm{Re}\{Z_s\}}$$

$$= \frac{18.7^2}{8\cdot 3.42}$$

$$= \boxed{12.8\,W}$$