Obj Phasors

- Reviewing cos + sin $\leftrightarrow$ cos, θ
- Intuition
- Math
- Calculator
- Applications

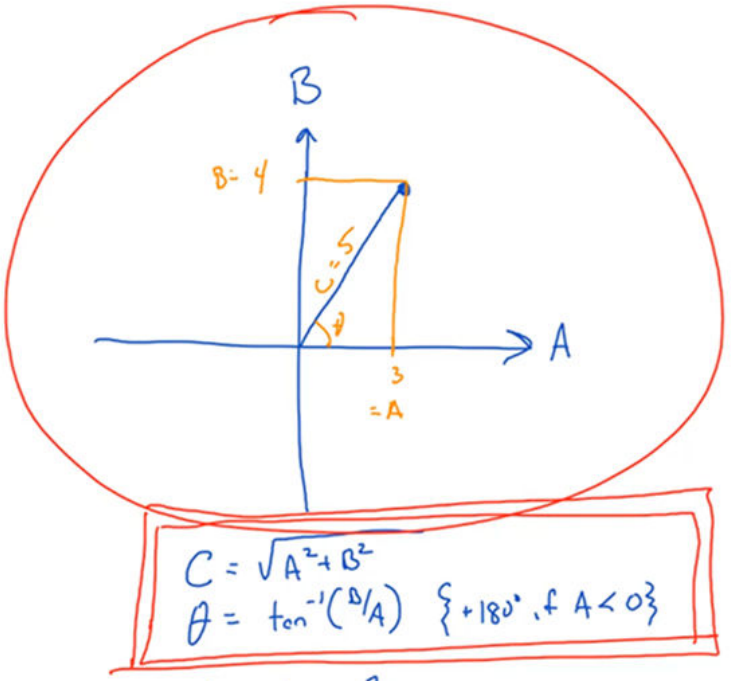
Last Time rectangular

$A\cos(\omega t) - B\sin(\omega t) \iff C\cos(\omega t + \theta)$ polar

Ex $3\cos 7t - 4\sin 7t = 5\cos(\omega t + 53°)$

$A = 3$

$B = 4$



$C = \sqrt{A^2 + B^2}$

$\theta = \tan^{-1}(B/A)$ {+180°, if $A < 0$}

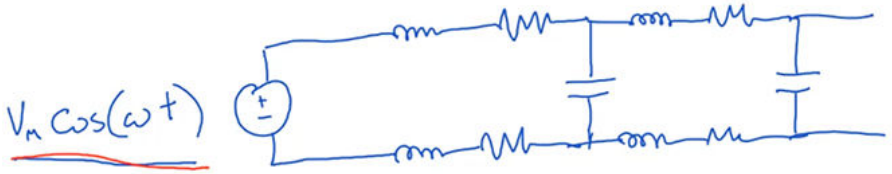
$A = C\cos\theta$

$B = C\sin\theta$

Ex $3\cos 7t - 4\sin 7t$ $\quad 5\cos(7t + 53°)$

$5\cos(14t + 53°)$

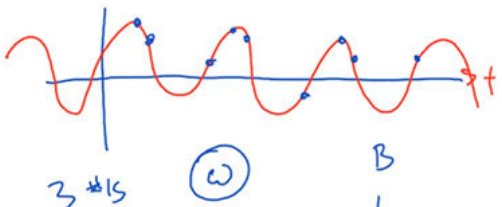
Historical Perspective

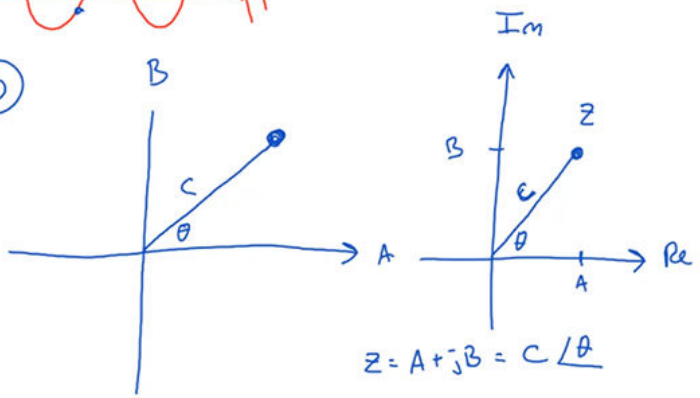


- No ICs!
- Sinusoidal Steady State (SSS)
- Only forced soln

Phasors: representation of sinusoid as a single complex number (at ω)

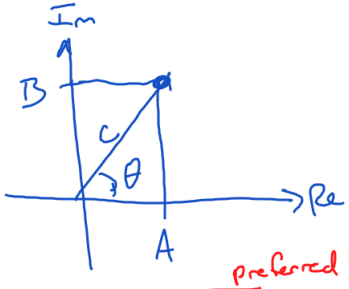
represent sinusoid ⇒ time domain ∞ # of points





$Z = A + jB = C \angle \theta$

Complex Math



1. Notation

- Rect. $A+jB$
- Polar $C\angle\theta$ ← radians
- Exponential $Ce^{j\theta}$ ← radians

preferred

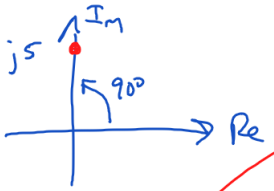
$C\angle\theta^\circ$ ← degrees

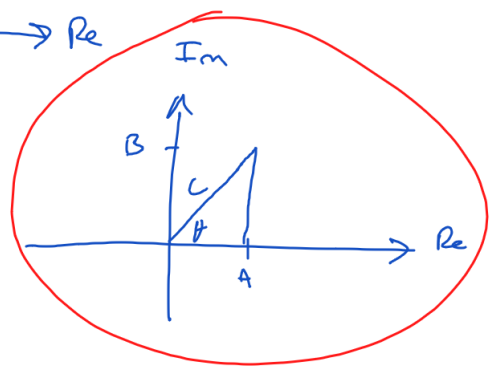
- $Ce^{j\theta} = \underbrace{C\cos\theta}_{\text{real}} + j\underbrace{C\sin\theta}_{\text{imag}}$

Ex $5\angle 90^\circ = 5\angle \pi/2$

MATLAB: `>> 5 * exp(j*pi/2)`

`>> j5`





2. Converting b/n forms

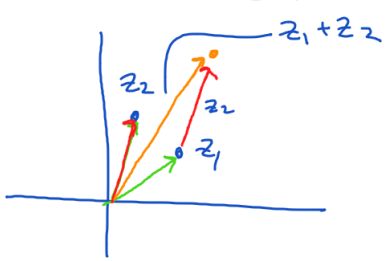
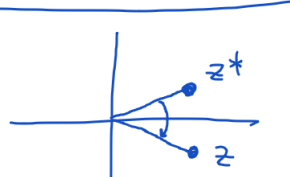
$A = C\cos\theta$

$B = C\sin\theta$

$C = \sqrt{A^2+B^2}$

$\theta = \tan^{-1}\left(\frac{B}{A}\right)$ {+180° if A < 0}

3. Math

	Rectangular	Polar	Graphical	
Sum	$\begin{array}{r} 2+j3 \\ +\ 4-j6 \\ \hline 6-j3 \end{array}$	hard $2\angle 13° + 6\angle -7°$		
mult	$\begin{array}{r} 2+j3 \\ \times\ 4-j6 \\ \hline (8+18)+j(-12+12) \\ =26 \end{array}$	easy $(2\angle 13°)(6\angle -7°)$ $12\angle 6°$	hard	$j=\sqrt{-1}$ $j^2=-1$ $-j^2=1$
divide	hard $\frac{(2+j3)(4+j6)}{(4-j6)(4+j6)} =$ ___	$\frac{6\angle 20°}{2\angle 10°} = 3\angle^{20°-10°}_{=10°}$	hard	
Complex Conj	$(2+j3)^*$ $=2-j3$	$(6\angle 20°)^* = 6\angle -20°$		

Calculator

ex $\dfrac{2e^{j\frac{\pi}{4}} - 4\angle 30^{\circ}}{6+j8}$ (rad, degrees)

degrees

- Put calc mode { degree / Complex polar
- $\dfrac{2\angle 45^{\circ} - 4\angle 30^{\circ}}{6+j8}$
- $((2\angle 45) - (4\angle 30))/(6+i\,8)$

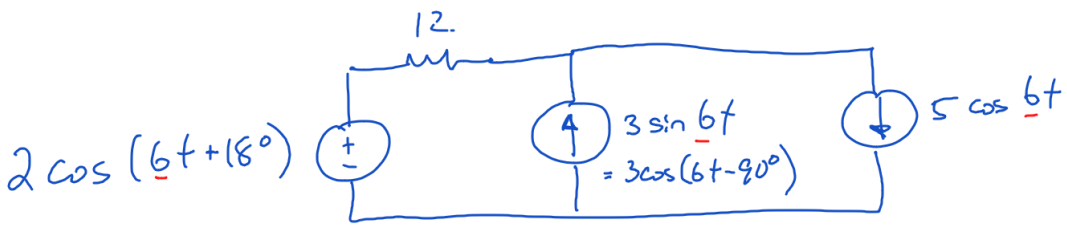
rad

- Put calc mode { radians / complex polar
- $\dfrac{2e^{j\pi/4} - 4e^{j\pi/6}}{6+j8}$
- $(2\exp(i\,\pi/4) - 4\exp(i\,\pi/6))/(6+i\,8)$

Domain error?

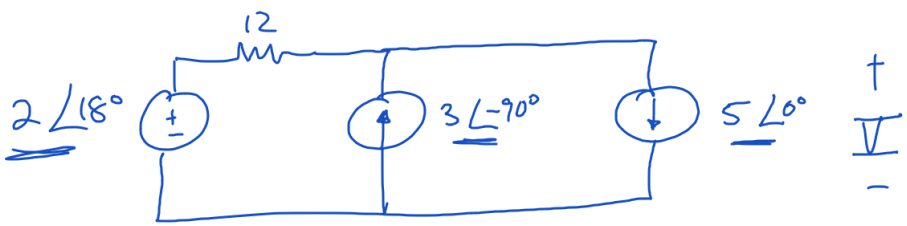
Wrong mode

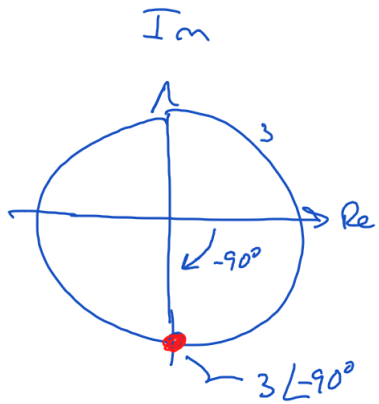
Applications magic!

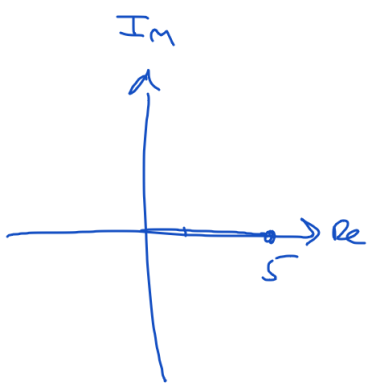


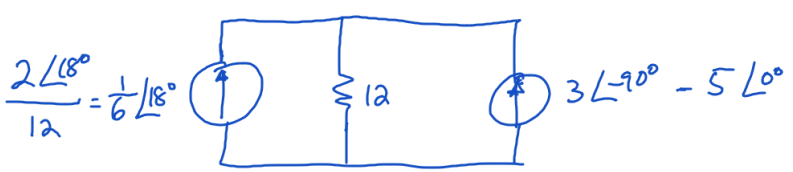
$3\sin(6t) = 3\cos(6t-90°)$

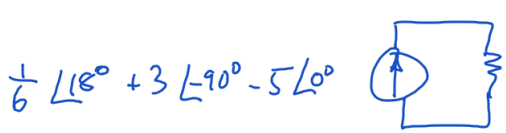
$\omega = 6$ Phasor Form











$$V = 12\left(\tfrac{1}{6}\angle 18° + \underbrace{3\angle -90°}_{j3} - \underbrace{5\angle 0°}_{5}\right)$$

$$= 68\angle -148°$$

$$v(t) = 68\cos(6t - 148°)\ V$$

- calc mode degree, complex polar
- 12(((1/6)∠18) + i 3 - 5)