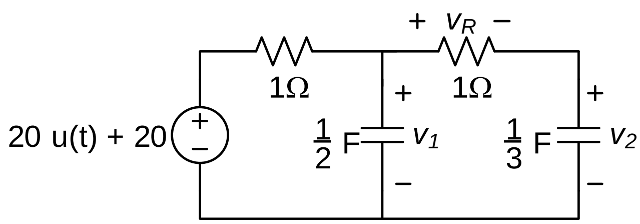
You have a large selection of collaborative problems, homework problems, and solved problems in your text already that are at the level of difficulty of the test. When you are ready for a challenge, here are three problems at a difficulty level slightly greater than that of the test.

P1 **Concept:** Initial Conditions

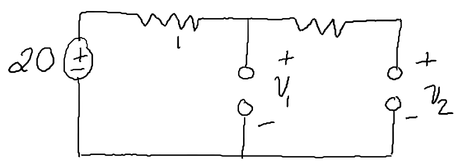
Find: $v_R(0^+)$ and $v_R'(0^+)$ in the following circuit

Hints: RCC initial conditions are solved the same way as LRC, except that instead of finding $v_C(0)$ and $i_L(0)$ first you'll find $v_{C1}(0)$ and $v_{C2}(0)$.



Find $v_R(0^+)$, $v_R'(0^+)$

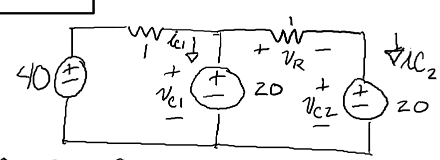
$t<0$



By inspection $v_1 = v_2 = 20V$

(No indicator, so find the IC's of the C's)

$t=0$



$v_R = 20 - 20 = 0V$

$i_{c1} = C v_{c1}'$

$\Rightarrow v_{c1}' = \frac{1}{C_1} i_{c1} = (2)(20) = 40V/s$

$v_{c2}' = \frac{1}{C_2} i_{c2} = (3)(0) = 0V/s$

KVL right Loop: $+v_R = -v_2 + v$

derivative

$v_R' = -v_2' + v'$

$= -0 + 40'$

$= 40 V/s$

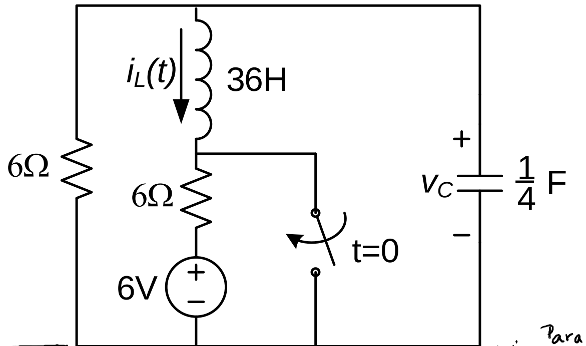
KCL top middle: $\frac{20-40}{1} + i_{c1} + \frac{20-20}{1} = 0$

$\Rightarrow \boxed{i_{c1} = 20A}$

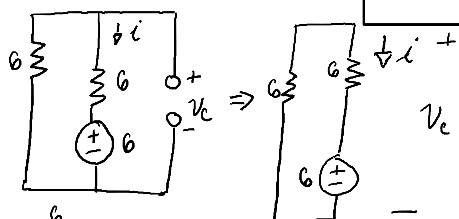
KCL top Right: $\frac{20-20}{1} = \boxed{i_{c2} = 0}$

So $\boxed{v_R(0^+) = 0V \quad v_R'(0^+) = 40 V/s}$

P2 **Concept:** Find *i* (parallel) or *v* (series) of an RLC circuit
Find: Find $v_C(t)$ in the following RLC circuit for $t \geq 0$.
Hints: There are several 3's in the answer



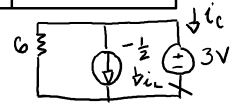
①



$i_{L0} = -\frac{6}{6+6}$ by Ω's law $= -\frac{1}{2}$ A

$v_{C0} = (6V)\left(\frac{6\Omega}{6\Omega+6\Omega}\right) = 3V$ by voltage divider

t = 0+



Parallel, so $v_o = v_c = 3V$

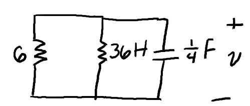
$v_o' = \frac{1}{C} i_c$

KVC top: $\frac{3}{6} - \frac{1}{2} + i_c = 0$

$\Rightarrow i_c = 0A$

$\Rightarrow v_o' = \frac{1}{c} i_c = 0$

t > 0



$\Rightarrow$ $v_o = 3V$, $v_o' = 0 V/s$

② $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$, $\alpha = \frac{1}{2RC} = \frac{1}{3}$ } Critically damped

③ $S = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -\frac{1}{3}, -\frac{1}{3} \Rightarrow$ $v_n(t) = C_1 e^{-\frac{1}{3}t} + C_2 t e^{-\frac{1}{3}t}$

④ $t = \infty$ no sources $\Rightarrow v_F = 0$

⑤ $v(t) = v_n(t) + v_f(t) = C_1 e^{-t/3} + C_2 t e^{-t/3}$ now match IC

$v(0) = C_1 = 3$ so

$v(t) = 3e^{-t/3} + C_2 t\, e^{-t/3}$

$v'(t) = -e^{-t/3} + C_2 t(-\frac{1}{3})e^{-t/3} + C_2 e^{-t/3}$ by product rule

$v'(0) = -1 + C_2(0) + C_2 \Rightarrow 0 = -1 + C_2 \Rightarrow C_2 = 1$

$v(t) = 3e^{-t/3} + t e^{-t/3}$

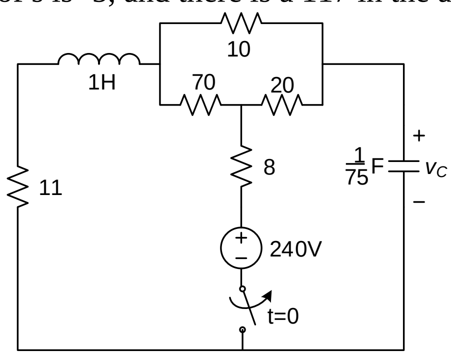
⑥

$$v(t) = \begin{cases} 3, & t < 0 \\ (3+t)e^{-t/3}, & t \geq 0 \end{cases}$$

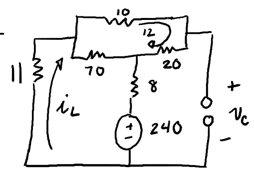
P3 Concept: Find any *i* or *v* in an RLC circuit

Find: Find $v_C(t)$ in the following RLC circuit for $t \geq 0$.

Hints: One root of s is -5, and there is a 117 in the answer



t < 0



Mesh:

i_L: $11 i_L + 70(i_L - i_2) + 8 i_L + 240 = 0$

$\Rightarrow 89 i_L - 70 i_2 = -240$

i_2: $10 i_2 + 20 i_2 + 70(i_2 - i_L) = 0$

$\Rightarrow -70 i_L + 100 i_2 = 0$

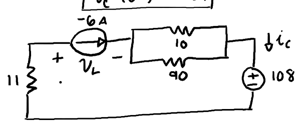
KVL Right loop: $-v_c + 20 i_2 + 8 i_L + 240 = 0$

$v_c = 20(-4.2) + 8(-6) + 240 = 108V$

$$\begin{bmatrix} 89 & -70 \\ -70 & 100 \end{bmatrix} \begin{bmatrix} i_L \\ i_2 \end{bmatrix} = \begin{bmatrix} -240 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} i_L \\ i_2 \end{bmatrix} = \begin{bmatrix} -6 \\ -4.2 \end{bmatrix}$$

so $i_L(0^+) = -6A$, $v_c(0^+) = 108V$

t = 0⁺



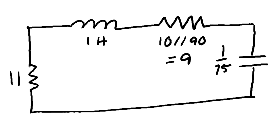
$\Rightarrow i_c = C v_c'$

$v_c' = \frac{1}{C} i_c = (75)(-6) = -450 \text{ V/s}$

$v_c(0^+) = 108V$

$v_c'(0^+) = -450 \text{ V/s}$

t > 0



$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1/75}} = \sqrt{75}$

$\alpha = \frac{R}{2L} = \frac{20}{2} = 10$

$\alpha > \omega_0 \Rightarrow$ overdamped

$s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -10 \pm 5 = -5, -15$

$v_n(t) = C_1 e^{-5t} + C_2 e^{-15t}$

t = ∞ No Source $\Rightarrow v_c(\infty) = 0 \Rightarrow v_f = 0$

$v(t) = v_n(t) + v_f(t) = C_1 e^{-5t} + C_2 e^{-15t}$

$v(0) = C_1 + C_2 = 108$

$v'(0) = -5C_1 - 15C_2 = -450$

$$\begin{bmatrix} 1 & 1 \\ -5 & -15 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 108 \\ -450 \end{bmatrix} \Rightarrow \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 117 \\ -9 \end{bmatrix}$$

$v(t) = 117 e^{-5t} - 9 e^{-15t} \text{ V}$