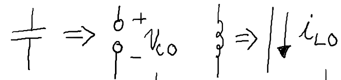
Given: 2nd Order Series or // RLC Circuit.

Find : X. X may be any Voltage or Current.

① Find IC

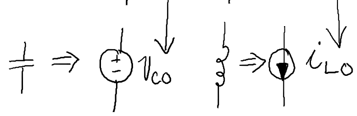
a) Draw $t<0$



no switches or $V(t)$'s

Find v_C, i_L

b) Draw $t=0^+$

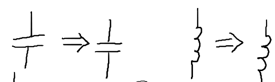


no switches or $V(t)$'s

Find X_0, X_0' by $\left\{\begin{matrix} i = Cv' \\ v = Li' \end{matrix}\right\}$ and maybe $\left\{\begin{matrix} V' = i'R & \Omega \text{ law} \\ V_1' + V_2' = 0 & KVL \\ i_1' + i_2' = 0 & KCL \end{matrix}\right.$

② Find natural Solution

a) Draw $t>0$



no switches or $v(t)$'s

Find $\omega_0 = \frac{1}{\sqrt{LC}}$, $\alpha = \frac{R}{2L}$ (series) or $\frac{1}{2RC}$ (parallel)

$S = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

b) Overdamped $\alpha > \omega_0$ $x_n = C_1 e^{S_1 t} + C_2 e^{S_2 t}$

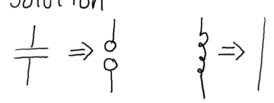
Critically damped $\alpha = \omega_0$ $x_n = C_1 e^{St} + C_2 t e^{St}$

Underdamped $\alpha < \omega_0$ $x_n = e^{-\alpha t}\left[C_1 \cos(\omega_d t) + C_2 \sin(\omega_d t)\right]$

↳ rewrite $S = -\alpha \pm j\omega_d$

③ Find Forced Solution

a) Draw $t=\infty$



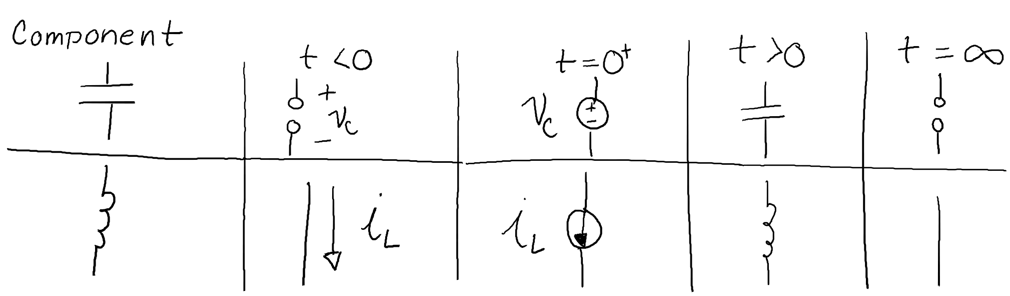
b) Find X_f

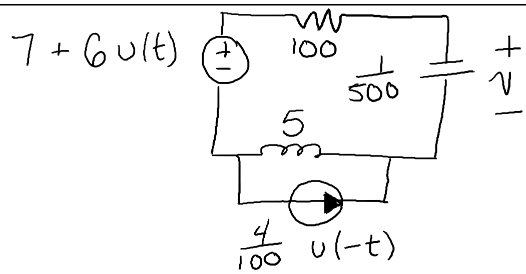
④ Find total response

a) $X(t) = X_n(t) + X_f(t)$

b) Solve for C_1 and C_2 in $X(t)$ using ICs

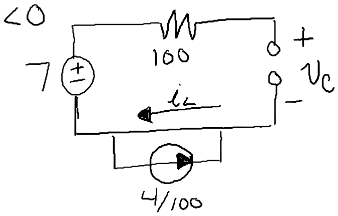
Summary





Find v

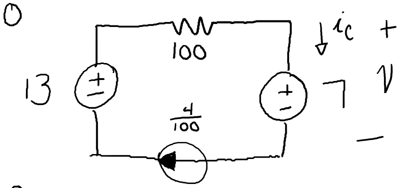
① $t<0$



$v_c = 7$

$i_L = \frac{4}{100}$

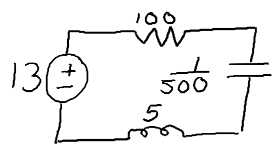
$t>0$



$v = 7$ by inspection

$i_c = C v_c' \Rightarrow v_c' = \frac{1}{C} i_c = 500\left(\frac{4}{100}\right) = 20$

② $t>0$



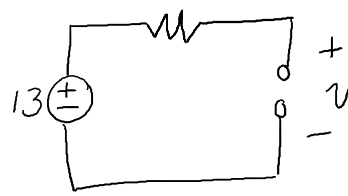
$\alpha = \frac{R}{2L} = \frac{100}{2 \cdot 5} = 10$

$\omega_0 = \frac{1}{\sqrt{LC}} = 10$

$\alpha = \omega_0 \Rightarrow$ Critically damped, $S = -10, -10$

$v_n = C_1 e^{-10t} + C_2 t e^{-10t}$

③ $t = \infty$



$v_f = 13$

④ $v(t) = v_n + v_f = C_1 e^{-10t} + C_2 t e^{-10t} + 13$

$v(0) = C_1 + 13 = 7$ by IC $\Rightarrow$ $\Rightarrow$ $C_1 = -6$

$v'(t) = -10 C_1 e^{-10t} + C_2 \left[t(-10) e^{-10t} + e^{-10t} \right]$

$v'(0) = -10C_1 + C_2 = 20$ by IC $\Rightarrow$ $C_2 = -40$

$$\boxed{v(t) = -6e^{-10t} - 40te^{-10t} + 13}$$