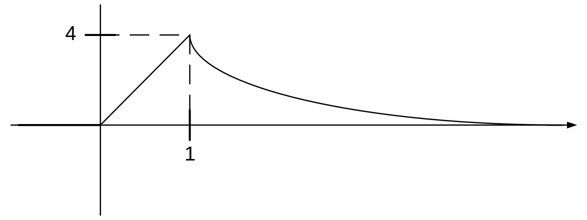
1. **Given** $v(t)=\begin{cases}0, & t\le 0,\\ 4t, & 0<t\le 1\\ 4e^{-(t-1)}, & t>1\end{cases}$

across a $\frac{1}{2}\mu F$ capacitor



a) **Find** $i(t)$ through it
b) **Find** $p(t)$ delivered to it (power)
c) **Find** $w(t)$ stored in it (energy)
d) **Find** $\int_0^\infty p(t)\ dt$ and comment on its significance

a) $i = Cv'$

$$= \begin{cases} 0\text{ A}, & t\le 0\\ (0.5\mu)(4) = 2\mu\text{A} & ; 0<t\le 1\\ (0.5\mu)(-4e^{-(t-1)}) = -2e^{-(t-1)}\mu\text{A} & , t>1\end{cases}$$

b) $p = i\cdot v$

$$= \begin{cases} 0\text{W} & , t\le 0\\ (4t)(2\mu) = 8t\mu\text{W} & ; 0<t\le 1\\ 4e^{-(t-1)}(-2e^{-(t-1)}\mu) = -8e^{-2(t-1)}\mu\text{W} & , t>1\end{cases}$$

c) $w = \frac{1}{2}Cv^2$

$$= \begin{cases} 0\text{ J} & , t\le 0\\ \frac{1}{2}\frac{1}{2}\mu(4t)^2 = 4t^2\mu\text{J} & ; 0<t\le 1\\ \frac{1}{2}\frac{1}{2}\mu(16e^{-2(t-1)}) = 4e^{-2(t-1)}\mu\text{J} & , t>1\end{cases}$$

d) Painful way: $\int_0^\infty p(t)\,dt = \int_0^1 p(t)\,dt + \int_1^\infty p(t)\,dt$

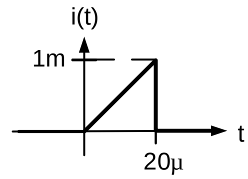
and substitute in values of $p(t)$ found in part (b). Ouch.

Smart way: $\int_0^\infty p(t)\,dt = w(\infty) - w(0)$ (integral of power is work)

$= 4e^{-200} - 0$

$= \boxed{0}$ much easier!

2. **Given** $i(t)=\begin{cases}0, & t\le 0\,\text{s},\\ 500t, & 0<t\le 20\mu s\\ 0, & t>20\mu s\end{cases}$



is delivered to an uncharged $0.2\mu F$ cap.

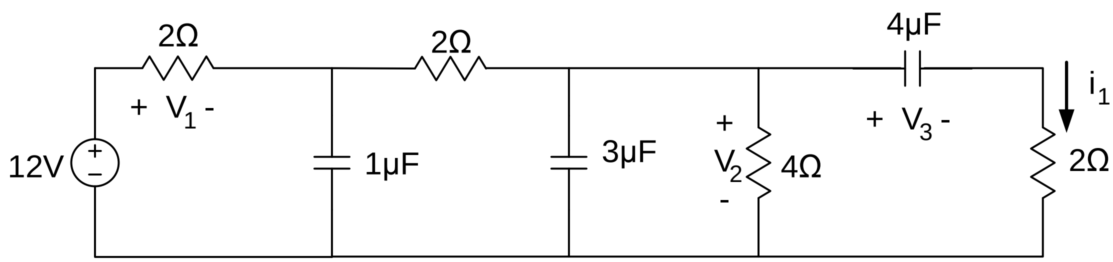
a) **Find** $v(t)$ across the capacitor for $t \ge 0$.
b) Why does a voltage remain on the capacitor after the current is zero from an intuitive viewpoint?

a)

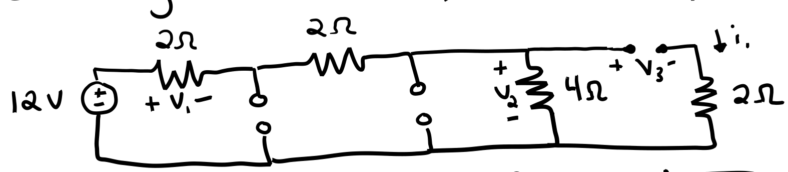
Region	t_0	$i(t)$	$v(t_0)$	$v(t)=\frac{1}{C}\int_{t_0}^{t} i(\tau)\,d\tau + v(t_0)$
$t<0$	$-\infty$	0	0	$=\frac{1}{C}\int 0\,d\tau + 0 = 0$
$0<t<20\mu$	0	$500t$	0	$=(5M)\int_0^t 500\,\tau\,d\tau + 0 = (5M)(250t^2) = 12.5t^2\,GV$
$t>20\mu$	20μ	0	$12.5(20\mu)^2 G = 12.5(400p)G = 5$	$= 0+5 = 5V$

b) After 20µS, 5V of "pressure" is stored in the stretched elastic "diaphram". With the current set to zero, no furthur movement of the diaphram is possible. So the voltage remains 5V.

3. **Given** the following circuit
Find v_1 , v_2 , v_3 , i_1



DC steady-state, so caps look like opens



By voltage divider, $V_1 = \frac{2}{2+2+4}12 =$ **3V**, $V_2 = \frac{4}{2+2+4}12 =$ **6V**

$i_1 = 0A$, and KVL around right loop: $-V_2 + V_3 + i_1 2 = 0 \Rightarrow$ **$V_3 = 6V$**