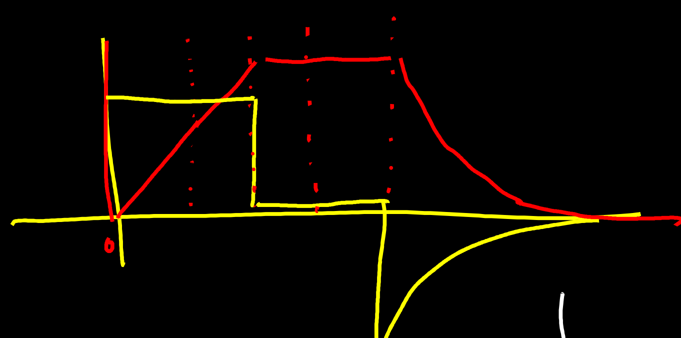
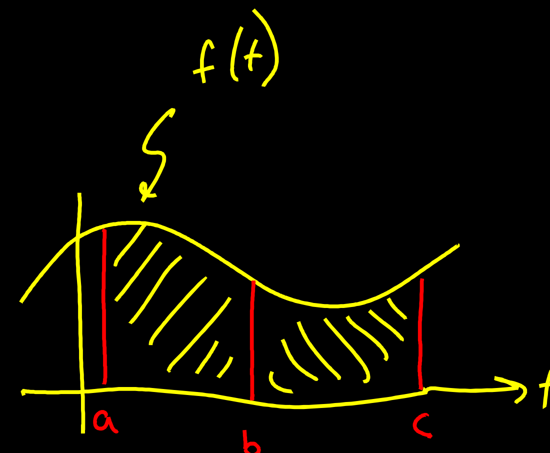
# Integrals

- **Notation**

$$\int_a^b v(t)\,dt$$

- **Intuition**




$$\int_a^c f(t)\,dt = \int_a^b f(t)\,dt + \int_b^c f(t)\,dt$$

- **Vs Derivatives**

$$\int v(t) = \int \frac{d}{dt} i(t)$$

$$i(t) = \int_{-\infty}^{t} v(\tau)\,d\tau$$

$i(0)$ $\quad v(t),\ t \geq 0$

$$i(t) = \int_{-\infty}^{t} v(\tau)\,d\tau = \boxed{\int_{-\infty}^{0} v(\tau)\,d\tau} + \int_0^t v(\tau)\,d\tau$$

$$i(0) = \boxed{\int_{-\infty}^{0} v(\tau)\,d\tau} \qquad i(t) = \underbrace{i(0)}_{I.C.} + \int_0^t v(\tau)\,d\tau$$

# Integrals vs. Derivatives

$$v(t) = \frac{d}{dt}\, i(t)$$

$$\boxed{i(t) = \int_{-\infty}^{t} v(\tau)\,d\tau}$$

$$v(t) = \begin{cases} 6e^{-2t}, & t \geq 0 \\ ?, & t < 0 \end{cases}$$

$$i(0) = 4$$

$$i(t) = \underbrace{\int_{-\infty}^{0} v(\tau)\,d\tau}_{i(0)} + \int_{0}^{t} v(\tau)\,d\tau$$

$$i(t) = i(0) + \int_0^t v(\tau)\,d\tau$$

$$= 4 + \int_0^t 6e^{-2\tau}\,d\tau$$

$$= 4 + 6(-2)e^{-2\tau}\Big]_{\tau=0}^{t}$$

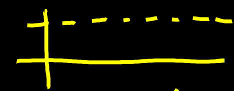
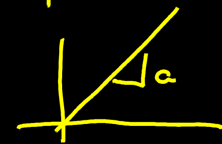
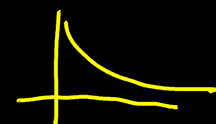
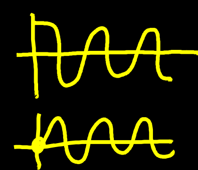
$$= 4 - 12\left[e^{-2t} - e^{-2\cdot 0}\right]$$

$$= 4 - 12\left[e^{-2t} - 1\right]$$

$$= \boxed{16 - 12e^{-2t}, \quad t \geq 0}$$

# Integrals

- **Common ones**

| $f(t)$ | | $\int f(t)\,dt$ |
|---|---|---|
| $C$ |  | $Ct$ |
| $at$ |  | $\frac{1}{2}at^2$ |
| $e^{-at}$ |  | $-\frac{1}{a}e^{-at}$ |
| $\cos(\omega t+\theta)$<br>$\sin(\omega t+\theta)$ |  | $\frac{1}{\omega}\sin(\omega t+\theta)$<br>$-\frac{1}{\omega}\cos(\omega t+\theta)$ |



- **Properties**

| | |
|---|---|
| $C \cdot f(t)$ $\quad 2e^{-3t}$ | $C\int f(t)\,dt$ $\quad 2(-\frac{1}{3})e^{-3t}$ |
| $f(t)+g(t)$ | $\int f(t)+\int g(t)$ |
| $f(t)\cdot g(t)$ | X hard! |